1. Schroeder 1.16

(a) The three forces acting on the slab of thickness $dz$ are gravity and the pressure from above and below. To achieve equilibrium they must add to zero.

$$0 = -mg \rho Adz - [P(z + dz) - P(z)]A$$  

(1)

Expanding $P(z)$ in a Taylor series yields

$$P(z + dz) = P(z) + \partial P/\partial z \times dz = P(z) + dP(z)$$

and after division by $Adz$ we thus find:

$$\frac{dP}{dz} = -mg \rho$$  

(2)

(b) The ideal gas law is $PV = Nk_BT \Rightarrow \rho = (mP)/(k_BT)$ inserting gives the barometric equation

$$\frac{dP}{dz} = -\frac{mg}{k_BT}P$$  

(3)

(c) For a height independent temperature we use an exponential ansatz $P(z) = P(0)e^{\lambda z}$ that yields

$$\frac{dP}{dz} = \lambda P(0)e^{\lambda z} = -\frac{mg}{k_BT}P(0)e^{\lambda z}$$  

(4)

This again gives $\lambda = -(mg)/(k_BT)$ and thus

$$P(z) = P(0)e^{-mgz/kT}$$  

(5)

Substituting this function for $P(z)$ into the formula for $\rho = mP/kT$ gives us

$$\rho(z) = \frac{mP(0)}{kT}e^{-mgz/kT} = \rho(0)e^{-mgz/kT}$$  

(6)

(d) The numerical value for $mg/kT$ is equal to $1.18 \times 10^{-4}/m$ for an average molecule mass 28.8$u$ and a temperature of 293$K$. Thus we find

<table>
<thead>
<tr>
<th>P / atm</th>
<th>Ogden, Utah</th>
<th>Leadville, Colorado</th>
<th>Mt. Whitney, California</th>
<th>Mt. Everest, Nepal/Tibet</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84</td>
<td>0.69</td>
<td>0.59</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

However we have to bear in mind that these values are approximate. Especially for the Mt. Everest we would find a very different value if we took the temperature decrease into account.

2. Schroeder 1.22

(a) Each molecule hitting the Area $A$ will on average exert a force of $2m\overline{v_x}$, dividing the total force by this amount we find an approximation for the average number of molecules hitting the wall.

$$\Delta N = \frac{PA}{2m\overline{v_x}}\Delta t$$  

(7)

(b) Using $(\overline{v^2})^{1/2}$ as an approximation to $\overline{v_x}$ we use $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2} = 3kT/m$ to find

$\overline{v_x} \approx \sqrt{kT/m}$. 

1
(c) Using the ideal gas law to substitute \( P \) and (b) we find

\[
\frac{dN}{dt} = -\frac{PA}{2m} = -\frac{A}{2V} \sqrt{\frac{kT}{m}} N
\]

Again an exponential ansatz \( N(t) = N(0)e^{-t/\tau} \) will lead us to the result

\[
-\frac{1}{\tau} N(t) = -\frac{A}{2V} \sqrt{\frac{kT}{m}} N(t) \Rightarrow \tau = \frac{2V}{A} \sqrt{\frac{m}{kT}}
\]

(d) Assuming the same data for air as in (1) we find the characteristic time \( \tau = 6.88s \).

(e) Assuming a diameter of 28" and a radius of one inch we find a volume of \( 4.53 \times 10^{-3}m^3 \). Estimating the pressure in the tire at the beginning of the leaking to be about 4 \( atm \) we can infer from the ideal gas law that the density inside the tire is four times higher than outside at sea level. We thus need that the number of molecules drops to a fourth.

\[
\frac{t}{\tau} = \log 4 \approx 1.38
\]

Now solving for the size of the hole we have

\[
A = \frac{2V}{\tau} \sqrt{\frac{m}{kT}} = \frac{2V \times 1.38}{t} \sqrt{\frac{m}{kT}} = 1.19 \times 10^{-2}mm^2
\]

(f) We make the assumptions that the window with the dog in it leaves about 1000 \( cm^2 \) uncovered, that the capsule might approximately have a volume of 20 \( m^3 \) and that it takes our adventurers about one second to get rid of the dog. Assuming air at room temperature in their capsule we find

\[
\tau = 13.75s \Rightarrow \frac{N_{\text{final}}}{N_{\text{initial}}} \approx e^{-1/13.7} = 0.39
\]

We might thus conclude that despite our rather positive assumptions this ingenious method is not a realistic way to get rid of the dog corpse. However it is not as bad as one might suspect.

3. Schroeder 1.31
(c) Knowing the ideal gas law we have $PV = NkT = (2/f)U$. In the case of Helium, a monatomic gas, $f = 3$. We have

$$U_i = \frac{3}{2}P_iV_i \Rightarrow U_f = \frac{3}{2}9P_iV_i = 9U_i$$

and thus $\Delta U = 8U_i = 12P_iV_i = 1215.6 J$.

(d) We know from energy conservation $\Delta U = Q + W$ and thus

$$Q = \Delta U - W = (12 - (-4))P_iV_i = 1620.8 J$$

(e) To realize a pressure rising proportional to the volume we need some device whose force increases linearly. A possible example is a cylinder with movable piston, if we connect the piston to a spring. The volume increase leads to a linear decrease in the length of the spring. This decrease via Hooke’s law to a linearly rising pressure, if all other walls of the cylinder are fixed.

4. Schroeder 1.32

![Graph](image)

The work needed is

$$W = -\int_{V_i}^{V_f} P(V) dV,$$

the area under the curve in the PV-diagram. In our case however the volume occupied by the water almost does not change at all. If we assume that the pressure rises linearly with the volume reduction the area under the curve is in good approximation a triangle.

$$W = \frac{1}{2} \times 0.01 \times 10^{-3} m^3 \times 200 \times 1.013 \times 10^{-5} N/m^2 \approx 1 J$$

This is indeed very small, but we need to remember that work is force multiplied by distance. Here the latter one is obviously very small.

5. Schroeder 1.34

(a) We can always use $\Delta U = Q + W = f/2 \times Nk\Delta T = f/2 \times V \Delta P$, where the latter equality follows at fixed volume from the ideal gas law. We find the degrees of freedom to be $3 + 2 = 5$, where we only have two rotational degrees of freedom as the rotation about the axis of the molecule is not excited for quantum mechanical reasons.

A There is no change in the volume, thus

$$\Delta U = 2.5V_1(P_2 - P_1) = Q$$
B The pressure is fixed, thus as $\Delta T = p/Nk \times \Delta V$

$$\Delta U = f/2 \times V \Delta V = 2.5P_2(V_2 - V_1)$$

(19)

and

$$W = -P_2 \Delta V = -P_2(V_2 - V_1)$$

(20)

and finally $Q = \Delta U - W = 3.5P_2(V_2 - V_1)$.

C Again no change in volume, thus

$$\Delta U = -2.5V_2(P_2 - P_1) = Q$$

(21)

D Again the pressure is fixed, thus

$$\Delta U = -2.5P_1(V_2 - V_1) \text{ and } W = P_1(V_2 - V_1)$$

(22)

and $Q = \Delta U - W = -3.5P_1(V_2 - V_1)$.

(b) During step A we fix the piston and heat the gas. In step B we keep the force on the piston constant, leading to an expansion of the gas while we still add some heat. In C we cool the gas until the force on the fixed piston is again $P_1$. Finally in step D we compress the gas, absorbing the amount of heat necessary to keep the pressure fixed.

(c)

$$\sum_{A,B,C,D} U_i = -2.5(V_2 - V_1)(P_2 - P_1) + 2.5(P_2 - P_1)(V_2 - V_1) = 0$$

(23)

This is expected, as the internal energy for an ideal gas is only determined by its temperature. But the temperature does not change under one whole cycle, as this means that the system returns to the initial values of the state variables $P$ and $V$.

$$\sum_{A,B,C,D} W_i = -(P_2 + P_1)(V_2 - V_1)$$

(24)

and from $0 = \Delta U = Q + W$ it follows $Q = (P_2 - P_1)(V_2 - V_1)$. We find that we have added heat to the system and the system in turn has performed work on the piston.

6. Schroeder 1.36

(a) Since, by assumption, the process is adiabatic, the quantity $PV^\gamma$ is conserved. For air we have $\gamma = (5 + 2)/5 = 1.4$.

$$P_1V_1^\gamma = 7P_1V_2^\gamma \Rightarrow V_2 = 0.249V_1 = 0.249l$$

(25)

(b) As no heat is exchanged, the work is equal to the change in internal energy.

$$W = 5/2Nk(T_2 - T_1) = 5/2(P_2V_2 - P_1V_1) = 5/2 \times 0.743 \times 101.3J = 188J$$

(26)

(c) From the ideal gas law we have that $P^{1-\gamma}T^\gamma$ is another conserved quantity. Thus

$$P_1^{-0.4}T_1^{1.4} = (7P_1)^{-0.4}T_2^{1.4} \Rightarrow T_2 = 7^{2/7}T_1 = 1.74T_1 = 522K$$

(27)
7. Schroeder 1.41

(a) To heat 250g of water by 4K we use \( Q = C_p \Delta T \) and assume that \( C_p \) is roughly constant and equal to 4.187 J/gK in this regime. Thus for our 250g we have \( C_p = 1047 J/K \). The amount of heat gained by the water is thus \( Q_{H_2O} = 4188 J \).

(b) As we assume that there is no heat exchange with the styrofoam cup or the atmosphere we know immediately that \( Q_{metal} = -4188 J \).

(c) We may assume that after a minute the metal is in thermal equilibrium with the water. Thus \( \Delta T = 76K \). Using the same formula as in (a) we determine \( C_p(\text{metal}) = 4188 J/76K = 55 J/K \).

(d) The metal used in this experiment has a specific heat capacity of 0.55 J/gK.

8. Schroeder 1.47

We add an amount of ice to absorb heat from the boiling tea so that ice and tea eventually reach thermal equilibrium at 65 degree Celsius. In this process the heat that can be absorbed by one gram of ice is

\[
\frac{Q}{g} = \frac{7.5 \text{ cal} + 80 \text{ cal} + 65 \text{ cal}}{g} = 152.5 \text{ cal/g}
\]

where the first contribution is the heat capacity of ice, the second one the latent heat and the last one the heat capacity of water. It is interesting to note that it takes almost as much heat to melt the ice as it takes to bring water to boiling. To cool our tea we need \( Q = -7000 \text{ cal} \). Since we assume that the system does not exchange heat with its surroundings, the total change in internal energy is zero. Obviously there is no work done either, thus

\[
0 = \Delta U = Q = x152.5 \text{ cal/g} + (-7000 \text{ cal}) \Rightarrow x = 45.9 g
\]