Furthermore, since the energy can be divided up any way, the kinetic energy of the $\beta^-$ can vary from zero up to approximately 0.5 MeV as long as it plus the kinetic energy of the $\bar{\nu}$ and the mass of the $\bar{\nu}$ add up to 0.5 MeV.

$$KE_{\beta^-} + KE_{\bar{\nu}} + m_{\bar{\nu}} = 0.5\text{MeV}$$

1.3 What do we know about the neutrino?

i. The charge of the neutrino is zero.

Charge must always be conserved. Since the charge of a neutron is zero, the sum of the charges of the decay products must add to zero as well. Since the charges of the proton and $\beta^-$ already add to zero, any extra particle must have zero charge as well.

ii. The mass of the neutrino is very small.

Consider the kinetic energy equation above and Fig. 1. The kinetic energy equation must be true at all times, and hence must be true for every kinetic energy measured for the $\beta^-$ in Fig. 1. Let us choose a simple case - when the neutrino has zero kinetic energy and the $\beta^-$ has the maximum possible kinetic energy. As you can see from Fig. 1, that max energy is about 0.5 Mev. If we plug these into the formula, we get

$$0.5\text{MeV} + 0\text{MeV} + m_{\nu} = 0.5\text{MeV} \quad \text{or} \quad m_{\nu} \approx 0.$$  

(The approximate is there because now we really need to think about how close that curve really gets to 0.5 MeV - there are physicists who have spent their careers on this.) Thus we see that the neutrino mass must be very small, if not zero.

iii. The neutrino is weakly interacting.

This just comes out of the fact that it is nigh unto impossible to actually detect a neutrino. Generally physicists know that they're there by the fact that there is some missing energy - just like in the preceding analysis.

iv. The new particle in $\beta^-$ decay is an anti-particle.

We know this because both the $\beta^-$ particle and the neutrino feel the fourth fundamental force - the weak nuclear force. They then must have some property that lets them feel this force. Physicists call particles that feel the weak force leptons and say that particles that are leptons carry a lepton number. Just like electrical charge, lepton number must be conserved. The neutron that decayed does not have lepton number, nor does the proton that resulted. Therefore the lepton numbers of the $\beta^-$ and neutrino must add to zero. We define the lepton number of the $\beta^-$ to be 1, therefore the lepton number of the neutrino is -1 - it is an anti-particle and we write it with a bar over the top of the symbol, $\bar{\nu}$.

2 Special Relativity

2.1 Galilean Relativity

Galilean relativity is the the relativity of our everyday lives. Although it isn't perfect, it works well enough that we can use it in most situations (when objects are moving at much less than the speed of light). There are two types of problems that one is likely to see. One is the desperado-on-the-train type problem and the other is the cars-on-the-highway type problem. They are formulated slightly differently. Reminder: Just like in special relativity, Galilean relativity requires that the reference frames be inertial - that is, that there be no acceleration in the problem. If this is satisfied, then all frames are equivalent.

2.1.1 Desperado-on-the-train

For this type of problem there is a frame of reference which moves with respect to (wrt) the frame of reference of the observer? There is also an object moving in the moving frame of reference. How to you figure out the velocity of the moving object wrt to the frame of the observer. In the case of Fig. 2., the observer is the sheriff, the moving frame of reference is the desperado and the object moving in the moving frame of reference is the bullet. Here is where the equation from the notes fits in:
Figure 3.

\[ v = v' + u \]

where

- \( v \) = the velocity of the object (bullet) as seen from the observer's (sheriff's) frame.
- \( u \) = the relative velocity of the moving frame wrt the observer's (sheriff's) frame - the velocity of the train.
- \( v' \) = the velocity of the moving object (bullet) as seen in the moving frame (the train) - the muzzle velocity of the bullet.

2.1.2 Cars-on-the-highway

This really isn't different from 2.1.1, but the connection may be difficult to see. Take a look at Fig. 3. The observer is in a car heading North. If we wish to calculate, say, the velocity of car 1 wrt the observer, how do we identify which velocities are \( u \) and \( v' \)? It is clear (I hope!) that \( v \) is the velocity of 1 wrt the observer just like it was the velocity of the bullet wrt the sheriff in the above example. Here is the trick. In what frame is the velocity of car 1 defined? The frame of the ground. Is the ground moving wrt to the observer? Yes. Therefore the moving frame is the frame of the ground and the 'moving object' is car 1. OK, that means that \( v' = v_1 \). What is the velocity of the ground wrt the observer? This is just the negative of the velocity of the observer wrt the ground, so \( u = -v_0 \). Now, you can use the above formula to get

\[ v = u + v' = -v_0 + v_1. \]

(Note that since \( v_1 \) is heading south, it will be negative.) You can do the same thing for the relative velocity of car 2 wrt the observer, yielding

\[ v = u + v' = -v_0 + v_2. \]

2.2 Michelson-Morley – A Hint of Special Relativity

2.2.1 Waves

Waves always move at the same speed through any given medium. That means that sound always moves at the same speed through air independent of the movement of the source. (We'll get back to this later.) So the only way to see a wave moving faster or slower than that natural speed determined by the medium is to move wrt to the medium. Consider the example of the airplane and the control tower. The natural speed of sound wrt the air is about 1000 ft/s. The airplane measures the speed of the sound to be 1200 ft/s. How fast is the airplane moving wrt the air?

Here the observer is the guy on the airplane and the thing that he measures is the speed of the sound, so the measured speed of the sound is \( v \). The moving frame is the frame of the air, so the speed wrt the air is \( u \). Finally, we know the speed of the sound wrt the air - the air is the 'moving object' and so the speed of the sound is \( v' \). Then we have

\[ v = -1200 \text{ ft/s} = u + v' = u + (-1000 \text{ ft/s}). \]

The negatives are there because I am defining the direction in which the airplane is heading to be positive. Solving the above yields, \( u = -200 \text{ ft/s} \), or the air is moving 200 ft/s toward the airplane from the front.

2.2.2 Boats in a stream

The easiest way to understand the Michelson-Morley experiment is to start by understanding the motion of other objects moving up and down and cross-stream through a medium. Consider the boats below. Both move at the speed \( v \) wrt the water. The water is flowing down with a speed \( u \). The boats travel from their starting places to the black dots which are each a distance \( L \) from the starting point. The question is which one comes back first.
For boat 1 the speed that it moves upstream is \( v - u \) and the speed with which it moves downstream on its way back is \( v + u \). We use \( t = \frac{d}{v} \) to find the times to go up and down.

\[
T_{\text{total}} = T_{\text{up}} + T_{\text{down}}
= \frac{L}{v - u} + \frac{L}{v + u}
= \frac{2Lv}{v^2 - u^2}
\]

To find the time for the boat 2, we must first realize that in order to get to the point directly across the stream the boat needs to head slightly upstream. If the boat heads directly across the stream, the boat will be swept downstream and won't get to the dot. How far does it need to head upstream? It needs to head for a point that is \( uT \) upstream where \( T \) is the time that it takes to go across the river; one way. Then the distance that it travels is the hypotenuse \( L' \). Using this and \( t = \frac{d}{v} \)

\[
T = \frac{L'}{v}
\]

\[
v^2T^2 = L'^2 = L^2 + u^2T^2
\]

\[
(u^2 - v^2)T^2 = L^2
\]

\[
T = \frac{L}{\sqrt{v^2 - u^2}} = \frac{L}{u\sqrt{1 - u^2/v^2}}
\]

\[
T_{\text{total}} = 2T = \frac{2L}{v\sqrt{1 - u^2/v^2}}
\]

Now, the typical person probably fell asleep somewhere in the middle of all that algebra. The important result of all that was that the time it takes to go up and down stream is longer than the time to go across and back. One can see that this is true because both denominators have \( 1 - u^2/v^2 \) which is less than 1, but the square root of a number less than one is larger than the original number, so

\[
1 - u^2/v^2 < \sqrt{1 - u^2/v^2}
\]

If this is true then \( T_{\text{total}} \) for boat 2 must be smaller.

Boats (and waves) traveling up and downstream take longer to return than boats (and waves) traveling across stream and back.

2.3.3 The Michelson-Morley Experiment

Since Maxwell showed that light was an electromagnetic wave, it was thought that like any other wave, light must move through a medium. That medium was called the ether. The ether must pervade all space (at least what we could see of it!) since it was clear that we were getting light from faraway stars and that that light had to travel through something. Michelson and Morley built an experiment to measure the velocity of the earth through the ether. Their apparatus used principles similar to that in the boat example above. One light beam would travel out to a mirror and bounce back while a second light beam would do the same in a direction \( 90^\circ \) from from the first light beam. (See Fig. 5.) If the theory of the ether were correct they would find a time lag and the slower direction would be the direction of the movement of the ether.

Three explanations were given to account for the results:

- The earth and ether aren't moving with respect to each other.

This was obviously false because the earth changes directions as it circles the sun. Thus, even if it was momentarily at rest wrt the ether, it would not be six months later. Since the experiment yielded the same results whenever it was done, this could not be the explanation.

- Lorentz Contraction

This was an ad hoc rule which proposed that the length of the apparatus that was parallel to the flow of the ether actually shrank. Then, although the average speed of the light beam would be less, the length of the apparatus would be proportionally shorter so that the overall, round-trip time would be the same for the two light beams. As special relativity will show later, this was also incorrect.
3.3 The Postulates of Relativity

i. All inertial reference frames are equivalent.

ii. The speed of light is independent of the motion of the observer or of the source.

Comments:
First of all, realize that a postulate is something that you just have to accept until such time as it is proven wrong. There are no proofs.

All of the special relativity we will talk about involves inertial reference frames – that is, reference frames that are in no way accelerated. They don't speed up, they don't slow down, and they don't change direction.

2.4 The three weirdnesses of Special Relativity

2.4.1 Simultaneity Breakdown

Yikes! This is by far the weirdest. The idea is this:

Events that are not causally related can happen in different sequences depending on the reference frame of the observer.

Mind you, this is not because of any failing of the observer to properly observe the sequence of events. In his frame the events have a certain order, in another frame the events have a different order. This is not perception – this is reality.

Here is the example of simultaneity breakdown that was given in class. There are two space ships moving past each other as shown in Fig. 6. Each moves at speed \(v\). In the center of each spaceship is an observer. Just as the two spaceships pull alongside each other a pair of asteroids hits them on each end of their spaceships. When the impacts occur, a signal is sent from each end of the spaceship to the observer in the middle.

Let us say that observer A sees the signals from the impacts arrive at his location at exactly the same time. He knows the distance from his location to the ends of his ship and since he believes that he is stationary wrt the location of the events, he says that he knows the distance that the signals had to travel and that those distances are equal. Since the distances the signals travel are equal, he believes that the events happened at the same time. Observer A radios observer B and tells him that he has had a simultaneous impact on either end of his ship.
Observer B laughts scornfully and says that, no, the impacts didn’t happen simultaneously because he could clearly see that observer A’s ship was moving at the time. (Observer B says that his ship is stationary and that observer A’s ship is moving.) B says that certainly A got the signals at the same time, but since he was moving, in the time that it took the signals to travel to A’s location, A had actually moved a little closer to the point at which event 2 occurred. If this were the case, then the signal from event 1 would have farther to travel than the signal from event 2. Furthermore, if the two signals arrived at A at the same time, then the signal from event 1 I would have had to have started out earlier in order to make it there on time. Thus, event 2 must have occurred before event 1. Beyond this, B says, I am standing still and I saw the signal from event 1 first, therefore I know that event 1 happened before event 2. Nyah, nyah.

Who is right? Both are equally right. After all they are both in inertial frames and the first Postulate of Relativity says that all inertial frames are equivalent. It is just that the observers in different frames can have events happen in different orders.

We are left with a couple of facts which summarize this example:

1. A and B can each claim that he is at rest in his own frame.
2. A and B only observe the events when the light signals reach them from the events.
3. It takes time for the light to reach the observer from the event.
4. During that time, from one observer’s point of view, the other observer has moved into one of the other signals and away from the other.

2.4.2 Time Dilation

Consider the light clock experiment below. In case a. two vertical mirrors are set up so that a photon reflects back and forth between them. The distance between the two mirrors is $L$ and the time for the light to complete one full cycle back and forth is simply $2L/c$ (where $c$ is the speed of light). We call this a light clock because it has a well defined period and so, if we chose to, we could use it as a clock. Now, in case b. the same light clock is moving past us (or we are moving past it...) at speed $v$ as shown. Because the pair of mirrors is traveling wrt us, we see the light beams trace out a triangular path as indicated. This means, however, that the path we see the light taking is longer than the path it took when the light clock was standing still wrt us!

(Remember that the hypotenuse of a right triangle is longer than the other sides of the triangle.) Since the speed of light is constant for all observers and the light travels farther, this means that the time we measure for the light clock to complete one cycle must be longer! Thus, the light clock is running slower than it was when it was standing still.

We can calculate how much slower it must be running. Let $T$ be the time for a full cycle that we measure when the clock is moving wrt us and $T_0$ is the time for a full cycle when the clock is at rest wrt us.

$$T_0 = \frac{2L}{c} \text{ from above}$$

$$T = \frac{2L’}{c}$$

$$T^2 = \frac{4}{c^2}L'^2 = \frac{4}{c^2} (L^2 + v^2T^2/c^2) = \left(\frac{2L}{c}\right)^2 + \frac{v^2}{c^2}T^2$$

$$T^2\left(1 - \frac{v^2}{c^2}\right) = T_0^2$$

$$T = T_0/\sqrt{1 - \frac{v^2}{c^2}}$$

If we define $\beta \equiv \frac{v}{c}$ and $\gamma \equiv 1/\sqrt{1 - \beta^2}$ then we may rewrite this as

$$T = \gamma T_0$$
Note that for nonzero \( \nu \), \( \beta \) is always less than one and \( \gamma \) is always greater than one. Remembering this will help you check your calculations. Also since \( \gamma \) is greater than one, \( T \) must be greater than \( T_0 \). This brings us to the basic rule for time dilation:

\[
\text{Moving clocks run slow.}
\]

\[2.4.3\] Length Contraction

There is a proof for this that is similar to that for time dilation, but Prof. did not give it to the class. If you really want to see it come talk to me. Instead we have an argument for the existence of length contraction.

Consider a particle called a muon. Don't worry what a muon is - all you need to know is that it is quite unstable. It decays in \( 2 \times 10^{-6} \) seconds. Let's say that the muon is formed at the top of Mt. Everest. Furthermore, it has a velocity very close to the speed of light, and it is headed toward the center of the earth. Just as the muon gets to the foot of the mountain, it decays. To an observer on the ground (at rest wrt the mountain) that mountain is 5.5 miles high. See Fig. 8.

The observer sees that the muon doesn't decay until it reaches the foot of the mountain, and he calculates that the time that it must have taken the muon to travel that distance is \( t = 5.5 \text{mi}/c = 5.5 \text{mi}/1.86 \times 10^8 \text{mi/s} = 30 \times 10^{-8} \text{ s} \). That is 15 times the lifetime of the muon! However, our insightful observer knows that moving clocks run slow and thinks nothing of it.

Now consider what is happening from the point of view of the muon. The muon can decide that it is at rest and that the mountain is swishing past it at nearly the speed of light. Well, according to the muon it only takes it \( 2 \times 10^{-8} \) s to go past the mountain (its lifetime). It then applies \( d = vt \) to determine how tall Mt. Everest is. (Smart particle) What it gets is \( d = 2 \times 10^{-8} \text{ s}, 1.86 \times 10^8 \text{ mi/s} = 0.37 \text{ mi} \). Yikes! The muon sees the mountain as being much shorter than the observer sees it to be! In fact, it sees it as being about 15 times shorter. Not-so-coincidentally, it turns out that \( \gamma \approx 15 \) as well. This leads us to the relation between proper and laboratory lengths:

\[
L = L_0/\gamma
\]

and the rule is:

\[
\text{Moving meter sticks are short.}
\]

\[2.5\] Which is which?

How do I keep track of which length is \( L \) and which is \( L_0 \)? (or \( T \) and \( T_0 \)?)

Remember this:

- All quantities with a zero subscript are the quantities that are measured when the measured thing is at rest with respect to the measurer. This is called the proper length/time/mass/etc.

- All quantities without a zero subscript are the quantities that are measured when the measured thing is moving wrt to the measurer. This is called the lab length/time/mass/etc.

For example, lets say I have a stick and you and I are moving wrt each other. The length of the stick that I measure is \( L_0 \) because I am at rest wrt the stick. The length that you measure is \( L \) because the stick is moving wrt you. Now I hand the stick to you. Now I measure \( L \) because the stick is moving wrt me and you measure \( L_0 \) because the stick is at rest wrt you.

\[2.6\] Lorentz Contraction Revisited

Recall that Lorentz suggested a contraction relation that was the same in form as the length contraction above. Does this mean that Lorentz was right? No, Lorentz wasn't right. What he suggested was that the effect of the ether caused matter to contract. What really happens is that space contracts. Thus, though his solution may have looked similar, the underlying idea was not physically relevant.
2.7 Relativistic Mass and Energy

There are two more relativistic quantities in which we are interested—mass and energy. Consider Einstein’s equation for mass and energy:

\[ E = mc^2 \]

This is true for all velocities, but we do not expect that energy is constant while velocity changes. We would expect the energy to increase as the object gains kinetic energy. It turns out that you can write the energy as follows:

\[ E = \gamma E_0 \]

where \( E_0 \) is the rest energy, which is further defined by

\[ E_0 = mc^2 \]

where \( m_0 \) is the rest mass. From the above we can derive that

\[ m = \gamma m_0 \]

2.8 Relativistic addition of velocities

OK, so you can’t add two velocities and get something larger than \( c \). How do you add large velocities? If I wish to add, say, \( u \) and \( u' \) to get a resulting velocity \( v \), I would do it as follows:

\[ v = \frac{u + u'}{1 + \frac{uu'}{c^2}} \]

We can check that this works when one of the velocities is \( c \).

\[ v = \frac{u + c}{1 + \frac{uu}{c^2}} = \frac{c + u}{c + u} = c \]

So this relation doesn’t violate the second Postulate of Relativity like the Galilean addition of velocities did.

2.9 The Relativistic Doppler effect

First of all, what is the Doppler effect? Basically, when the source of a signal moves toward you (or you move toward it) the frequency of the signal increases—this is called a blue-shift. Alternately, if the source is moving away from you (or you are moving away from it) the frequency decreases—this is called a red-shift.

Here is a simple example of a Doppler effect in water to give you an idea of how it works. See Fig. 9.

Imagine that I dip my finger into a pool of water every second. At \( t = 0 \) there is a small circular wave which grows to make a circle of ever increasing diameter, but the center of the circle does not move. Each time I dip my finger, a new circular wave starts. Now imagine that there is an observer in a small boat in the water. If it just sits and waits for the waves to form to come to him, he will see a frequency of one wave per second (the rate that I am dipping my finger). However if he moves toward the waves, (observer a.) he will see the waves come more often because he is heading into them (blue-shift). Now, if he heads away from the waves (observer b.) he will see the waves come less frequently because they have to catch up with him as he heads away (red-shift). This is the “moving observer” case.

Now imagine that I start to walk along as I dip my finger in the water. Now the center of each circle will be offset with respect to the last circle. This is the moving source case. Then the stationary observer who watches me move away (observer c.) will see the waves farther apart than they would normally be (red-shift), and the observer that watches me approach (observer d.) will see them come much more closely together (blue-shift).
The actual formulas for the frequency shifts are:

\[ f = f_0 \sqrt{\frac{1 + \beta}{1 - \beta}} \text{ approaching – blue-shift} \]
\[ f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}} \text{ receding – red-shift} \]

where \( f \) is the frequency measured by the observer moving wrt the source of the signal and \( f_0 \) is the frequency of the observer at rest wrt the source of the signal.

The Doppler effect has many uses, but one of the important ones for understanding the universe is its use in determining the velocity of far away stars. In the beginning of the class, we stated that one of the articles of faith of physics was that the nature of the universe is, well, universal. That is, that what goes on here in our neck of the woods is the same as what happens out in other galaxies. This would include that far away galaxies have the same elements that we do and that the atomic spectra of those elements is the same there as it is here. Thus, if we were to look at the light coming from distant stars we would see the atomic spectra for, say, H and He. However, if those stars are moving away (or toward) us the entire spectrum of each element would be shifted to lower (or higher) frequencies. If we measure that frequency shift and use our knowledge of the Doppler effect, we can calculate the speed at which those stars are receding (or approaching). In practice, we can see that most stars are receding from us - the universe is expanding.

2.10 The Pole Vaulter Paradox

A pole vaulter stands next to a barn (not moving wrt it) and sees that his pole is as long as the barn is wide. Another person stands in the barn as well. Now the pole vaulter starts running toward the barn at nearly the speed of light. He looks at the barn and thinks to himself, “Ah, the barn has shrunk due to length contraction, and if the person in the barn tries to close the front and back doors of the barn at the same time, I will not fit inside.” On the other hand, the person in the barn looks at the pole vaulter’s pole and says, “Ah, the pole has shrunk due to length contraction so I will have no problem fitting it in the barn when I close the front and back doors.” When the observer in the barn sees the pole vaulter enter the barn he closes the front and back doors at the same time, briefly trapping the pole vaulter inside. He then quickly opens them again so that the pole vaulter can run out the back of the barn. (How would you like to run into a barn door at nearly the speed of light? Ouch!) How can this have happened if, according to the pole vaulter, the pole won’t fit in the barn?

See Fig. 10. The answer is that we haven’t taken into account the breakdown of simultaneity. What the barn observer sees is the doors closing at the same time, but what the pole vaulter sees is the doors opening and closing at different times. Thus, as far as the vaulter is concerned, he is never trapped in the barn and it doesn’t matter that his pole is too long.

2.11 The Twin Paradox

The idea behind the twin paradox is that you have a pair of identical twins and one of them gets in a rocket and leaves the earth at a velocity close to the speed of light. After a while he turns around and comes back. According to the twin that stayed on earth, the rocket twin moved away very fast and then returned very fast. Since the rocket twin was moving all that time, the earth twin says that the rocket twin’s clock should have been running slow and so the rocket twin should be younger than the earth twin. However, the rocket twin says to the earth twin, “But I didn’t go anywhere! I saw the earth fly away from me at a speed near the speed of light and later return at nearly the speed of light. You have been moving all that time and so you should have had the slow clock and you should be the young one!” Well, at the end of the trip one of them is older than the other. What is the difference.
between the two of them that would determine who is the older?

The solution to the paradox is that we haven't taken into account the fact the one of the twins had to accelerate in order to leave the earth and come back later. We have said before that all inertial frames are equivalent, but as soon as one of the frames is not inertial, all bets are off. In this case it should be obvious to the twins which of them was accelerating. The accelerated one would feel all sorts of fictitious forces—like the ones that you feel when you round a corner in your car.

What are some good ways to think about why one twin is younger than the other?

2.11.1 Length Contraction

Let's say that the rocket twin is taking the rocket out to Altair (16 LY away) and back. If he goes fast enough, he will see the distance to Altair length contracted so that instead of seeing it as being 16 LY, he could instead see it as being, say, 1 LY. Then his round trip time would be about 2 years (since he is going at nearly the speed of light) whereas his earth twin would see it as taking 32 years. Then the earth twin would be 30 years older than the rocket twin when he got back.

2.11.2 Heartbeats (Doppler Effect)

This is pretty much and accounting method to let each of the twins keep track of how much the other twin has aged. Each of the twins agrees to send out at pulse every year so that the other can keep track of how old he is. How many does each count?

First, what the rocket twin receives: For the entire trip out to Altair, the rocket twin sees the earth twin's signal as red-shifted (lower frequency) because he is moving away from the earth twin. When the rocket twin turns around at Altair, he immediately starts heading into the earth twin's signal so he begins to see a blue-shifted signal (higher frequency). Therefore, for half the trip, the rocket twin receives the signal at a low frequency and for half the trip, he receives the signal at a high frequency.

Now, what the earth twin sees: This is a little more complicated. As the rocket twin speeds away from earth, he is giving out widely spaced signals because he is a moving source (See the Doppler effect section). All the way up to Altair the rocket twin gives out widely spaced signals. Then when he turns around at Altair, he immediately starts giving out signals that are closely packed together. So there is a blue-shift in the signal immediately.

However, that blue-shifted signal is still 16 LY away from the earth when the rocket twin starts making it. The earth twin must wait 16 years for the blue-shifted signal to make it back to the earth! Furthermore, since the rocket twin is moving at nearly the speed of light, he is following right behind those signals—according to the earth twin the speed of the light and of the rocket are so close that the light is only barely outrunning the rocket. The picture you might have is that the rocket is zooming back to earth with this growing packet of waves just in front of it. See Fig. 11. Then, since the packet of blue-shifted waves is only barely in front of the rocket, they all arrive at the earth just before the rocket twin does. Thus the earth twin gets approximately 32 years of red-shifted (low frequency) signal and a short time (a day, a week; it depends on exactly how fast the rocket goes) of blue-shifted signal (high frequency). Thus the idea is that since the earth twin measures a blue-shifted signal for a much shorter time than the rocket twin, the total number of signals he gets is smaller and the rocket twin must have aged less.
2.11.2 Accelerated Frames

One needs to be careful with the heartbeat argument. Even though the rocket twin may count his twin’s signals faster on the return trip, it does not mean that the earth twin is aging faster than the rocket twin is at that time. The rate of the signals is not the same as the rate of the aging! Here is why. While the rocket is moving at a steady pace toward Altair (as it might for a large fraction of the trip), the rocket twin will look back at his earth twin and say, “You are moving with me so your clock is slow and I am aging faster than you are.” The same is true for the steady velocity parts of the return trip. When does the earth twin age so much then? As we will find out in the next section, accelerating clocks run slow. The rocket twin must go through a tremendous amount of acceleration to get up to light speed, to turn around at Altair, and to slow down again once he is back at earth. During each of these periods, the rocket twin’s clock runs very slowly compared to the earth twin. Thus it is during these acceleration periods that the great difference in aging occurs.

3 General Relativity

Einstein’s Principle of Equivalence grows out of the surprising fact that to the best of our ability to measure it, inertial mass and gravitational mass are the same thing. By this I mean that the property of an object that causes it to resist changes in its motion (a = F/m) seems to be the same property that determines how that object responds to a gravitational field (F = mg). Don’t think that this is obvious. It is an amazing thing.

The equivalence principle states:

No experiment performed in one place can distinguish a gravitational force from an accelerated reference frame.

Here is an example from class which illustrates this. See Fig. 12. There are two identical elevators, each with an observer holding a ball. One elevator is in space and the other is on the earth. Initially, the elevator on earth is held in place with a cable. If the observer drops the ball, it falls to the floor due to gravity acting on it. At the same time, there is a force acting on the elevator in space. The force accelerates the elevator at exactly g. The elevator pushes on the observer inside and forces him upward. Since the elevator does not exert any force on the observer’s ball, once dropped, it falls to the floor in exactly the same way that the ball fell to the floor in the earth elevator. The two observers have no way of knowing which is on earth (as long as they stay in their elevators, at least!)

Next the cable holding up the earth observer’s elevator is cut. Now gravity pulls the elevator, the observer, and the dropped ball down at the same rate. Since everything is falling at the same rate, he and the ball just seem to float there. At the same time, we stop pulling on the space elevator so that the observer and his ball just float there, too. There is no way for the earth observer to tell himself from the space observer.

3.1 Consequences – Bending Light

One of the interesting consequences of the equivalence principle is that it predicts that gravity must bend light. Consider that you are in an elevator that is accelerating upward and someone shines a ray of light into the elevator. Since you are heading upward, you should see the light seem to dip down with respect to you. Since the equivalence principle states that whatever phenomena you see in an accelerated system must also happen in a system subjected to a gravitational force, you ought to see the light dip down when you are not accelerating but in a gravitational field instead. That is, the gravitational field must bend the light toward the source of the gravitational force.

This effect was confirmed in 1919 by Eddington who was able to see
starlight bend around the sun!

Also, what does this mean for the interaction of light with gravity? It suggests the light, although technically massless, has something that does allow it to feel gravitational force. Again Einstein gives us a clue with \( E = mc^2 \). Photons certainly have energy, and that energy is equivalent to mass. One upshot of all this is that, like massive objects, photons can have potential energy of a sort. That is, if a photon travels up out of a gravitational field (say, leaves the earth and goes into space) it gains potential energy. Well, if it gains potential energy it must get that energy from somewhere since energy must be conserved. This energy comes from a decrease in the photon's frequency. Since the frequency decreases, the effect is called a gravitational red-shift.

3.2 Consequences – Time Dilation from Acceleration

There's a problem with the frequency of light being greater on the surface of the earth than it is out in space. The number of wave cycles of the light that are sent down to the earth remain constant — you can't just magically add or subtract waves. So if all the waves that exist out in space actually make it to the earth, how can it be that we see them at a different rate? The answer is that if our clocks are running slow compared to the clocks in space, we will measure more cycles in our longer seconds. This will effectively give us a higher observed frequency.

Then, by the equivalence principle, if clocks run slow when there is a gravitational field, clocks must also run slow when there is acceleration. See the Twin Paradox section.

3.3 Consequences – Black Holes

So what happens if the photon doesn't have enough energy to get out of the gravitational field? This is the equivalent of me jumping up and down on the top of a hill and failing to launch myself into space. I simply don't have enough energy to do it. Instead I fall back to the ground. Likewise, the insufficiently energetic photon will fail to escape and will be trapped in the gravitational field. This is the basis for a black hole — a black hole is just a large mass that creates enough of a gravitational field to trap photons.

How do you see an object that traps photons? You have to do it indirectly. One way is to see how its large gravity affects nearby objects like stars. If you see a star orbiting, there must be something massive there to make it orbit. Secondly, charged particles can be trapped by the black hole as well. When these charged particles accelerate into the black hole, they radiate. Some of that light escapes and we can see that escaped light.