1. A spin $\frac{1}{2}$ particle is at rest in an oscillating magnetic field.

$$B = B_0 \cos(\omega t) \hat{k}$$

Construct the Hamiltonian matrix. At $t=0$ the system is in spin-up state with respect to the $x$-axis. Determine its state at any subsequent time.

Find the probability of getting $-\frac{1}{2}$ if you measure $S_x$ at any time $t$.

2. Spin $\frac{1}{2}$:
   1. Write down equations for the operators $S_x$, $S_y$ and $S_z$ for the spin 1/2 problem.
   2. Write down the two eigenstates of $S_z$.
   3. What is the eigenvalue of the operators $S^+$, $S_x+iS_y$ and $S^- = S_x-iS_y$ for each of these two states?

3. The protons in each water molecule in your body have spin $\frac{1}{2}$. When you enter a magnetic resonance imaging (MRI) scanner, each of your protons feels a magnetic field of $B=4$ Tesla along $z$: Assume there are $\sim 10^{27}$ protons per cubic meter in you.

   1. What is the Larmor precession frequency for protons at this field if the proton's magnetic moment is given by $\vec{\mu} = \gamma \vec{S}$; $\gamma = 10^{-26} \ (T s)^{-1}$
   2. Write down the most general spinor wavefunction for the proton spin describing this situation in terms of $\theta$, the angle that $\langle \hat{S} \rangle$ makes with the $z$-axis and $B$, $t$.
   3. If you measure the component of spin along the $x$-direction at time $t$, what is probability as a function of time that you will get $-\frac{\hbar}{2}$? You can think of this as the time dependence of the average $x$-component of spin.
   4. The $x$-component of "magnetization vector" $\vec{M}$ of all the proton spins is proportional to the average $x$-component magnetic moment

$$M_x = (N/V) \mu_x$$

where $N/V$ is the number per cubic meter of the protons. The $x$-component of magnetic field in Tesla due to this magnetization is

$$B_x = \mu_0 M_x; \mu_0 = 4\pi \times 10^{-7}$$.

Now, the voltage generated in a loop of wire of area $A$ which is being threaded by this time dependent $B_x(t)$ field is given by Faraday's Law:

$$V(t) = -A \frac{dB_x}{dt}.$$ 

What is the maximum voltage generated by you in the MRI if this pickup loop has
area $10^{-3} \text{ m}^2$ - hint: this occurs when $\alpha=90^\circ$.

4. An electron approaches a Stern-Gerlach (S-G) apparatus from the left and enters it at $t=0$. It is in the spin-up state.
   1. Write down its spinor wavefunction at the point of entry.
   2. The Hamiltonian for the spin $\frac{1}{2}$ electron is
      \[
      H = 0 \quad t < 0 \\
      = -\mu \cdot B = -\gamma (B_0 + \alpha z) S_z \quad \text{for } 0 < t < T \\
      = 0 \quad t > T
      \]
      where $T$ is the time for it to pass through the S-G apparatus, $B_0$ is the magnitude of the $B$-field along the $z$-axis and $\alpha$ is the magnitude of the $B$-field gradient. Write down the complete spinor wavefunction for the electron just at the point of exit of the apparatus.
   3. Considering only components in this wave function of the form $e^{ikz}$ write down the momentum along the $z$-direction which the electron picks up by virtue of passage through the S-G apparatus.
   4. What momentum would the electron have picked up if it were spin down upon entry?