1. Consider a 1-dimensional quantum simple harmonic oscillator (QSHO) with a particle of mass \( m \) and angular frequency \( \omega \).

Write down the Hamiltonian in terms of raising and lowering operators \( \hat{a}_+ \) and \( \hat{a}_- \).

At \( t=0 \)

\[ \Psi(x,0) = \sqrt{\frac{1}{3}} \psi_0(x) + \sqrt{\frac{2}{3}} \psi_2(x) \]

where \( \psi_n(x) \) are the stationary states of the QSHO.

What is \( \Psi(x,t) \)? What is the average value of energy of this state. Does it change with time?

2. Show that

\[
\int_{-\infty}^{\infty} [\hat{a}_+ \psi_n] dx = (n+1)\hbar \omega, \quad \int_{-\infty}^{\infty} [\hat{a}_- \psi_n] dx = (n)\hbar \omega.
\]

3. Calculate \( \langle x \rangle, \langle x^2 \rangle, \langle p \rangle, \langle p^2 \rangle \) for the stationary states \( \psi_0, \psi_1 \) of an QSHO.

4. Calculate kinetic energy expectation value \( \langle K \rangle \) and potential energy expectation value \( \langle V \rangle \) for the stationary states \( \psi_0, \psi_1 \) of an QSHO.

5. Find an operator \( N \) which counts the number of quanta in a given stationary state of the QSHO in terms of raising and lowering operators \( \hat{a}_+ \) and \( \hat{a}_- \).