1. Reading Assignment
Serway Sections 31.1–31.4, 32.1–32.3.

2. Serway 31.4A
In Figure P31.4 of your text find the current through section PQ, which has resistance $R$ and length $a$. The circuit is located in a magnetic field whose magnitude varies with time according to the expression $B = At$, where $A$ is a constant having units of teslas/second.

SOLUTION: The magnetic flux through each closed loop of the circuit is given by

\[ \Phi_{BL} = \oint B \cdot d\mathbf{a} = B \cdot (\text{Area}) = B(2a^2) = 2Aa^2, \]
\[ \Phi_{BR} = \oint B \cdot d\mathbf{a} = Aa^2. \]

The induced EMF in each loop can be found using Faraday’s Law of induction:

\[ \mathcal{E}_L = -\frac{d\Phi_{BL}}{dt} = -2Aa^2, \]
\[ \mathcal{E}_R = -\frac{d\Phi_{BR}}{dt} = -Aa^2. \]

Say there is a current $I_L$ clockwise around the left loop (which is the direction we expect by Lenz’s law if $A > 0$), and a current $I_R$ clockwise around the right loop. Then the current $I$ down through PQ is

\[ I = I_L - I_R. \]

These currents are related to the EMF’s around the loops by Ohm’s law:

\[ |\mathcal{E}_L| = 5RI_L + RI, \]
\[ |\mathcal{E}_R| = 3RI_L - RI, \]

where we have calculated the resistance in the part of the left loop not including PQ to be $5R$ since it has length $5a$ and we assume a uniform resistance per unit length of $R/a$ (as in the PQ wire); similarly for the right loop. Solving the above system of three linear equations for $I$, we find

\[ I = \frac{3|\mathcal{E}_L| - 5|\mathcal{E}_R|}{23R} = \frac{Aa^2}{23R}. \]

3. Serway 31.14
A long, straight wire carries a current $I = I_0 \sin(\omega t + \phi)$ and lies in the plane of a rectangular loop of $N$ turns of wire, as shown in Figure P31.14 of your text. The quantities $I_0$, $\omega$, and $\phi$ are all constants. Determine the emf induced in the loop by the magnetic field created by the current in the straight wire. Assume $I_0 = 50 \text{ A}$, $\omega = 200 \pi \text{ s}^{-1}$, $N = 100$, $a = b = 5 \text{ cm}$, and $\ell = 20 \text{ cm}$.

SOLUTION: From Ampere’s Law, we know that the magnitude of the magnetic field created by the long straight current carrying wire at a distance $x$ from the wire is

\[ B = \frac{\mu_0 I}{2\pi x}. \]

The magnetic flux through the rectangular loop of $N$ turns of wire is then given by:

\[ \Phi_B = N \cdot \oint B \cdot d\mathbf{a} = N \cdot \int_{a}^{b+a} \frac{\mu_0 I}{2\pi x} \ell \, dx = \frac{N \mu_0 I \ell}{2\pi} \ln \left( \frac{b+a}{a} \right) \]
\[ = \frac{N \mu_0 \ell}{2\pi} \ln \left( \frac{b+a}{a} \right) I_0 \sin(\omega t + \phi). \]
The EMF induced in the loop is then
\[ E = -\frac{d\Phi_B}{dt} = -\frac{N\mu_0\ell\omega}{2\pi} \ln \left( \frac{b+a}{a} \right) I_0 \cos(\omega t + \phi). \]

Plugging in \( I_0 = 50 \, \text{A}, \omega = 200\pi \, \text{s}^{-1}, N = 100, a = b = 5 \, \text{cm}, \) and \( l = 20 \, \text{cm}, \) the EMF is
\[ E = (0.09) \cos[(200\pi \, \text{s}^{-1})t + \phi] \, \text{V}. \]

4. Serway 31.27

Use Lenz’s law to answer the following questions concerning the direction of induced currents. (a) What is the direction of the induced current in resistor \( R \) in Figure P31.27a when the bar magnet is moved to the left? (b) What is the direction of the current induced in the resistor \( R \) right after the switch \( S \) in Figure P31.27b is closed? (c) What is the direction of the induced current in \( R \) when the current \( I \) in Figure P31.27c decreases rapidly to zero? (d) A copper bar is moved to the right while its axis is maintained perpendicularly to a magnetic field, as in Figure P31.27d. If the top of the bar becomes positive relative to the bottom, what is the direction of the magnetic field?

SOLUTION: Lenz’s Law states that the induced current and the induced EMF in a conductor are in such a direction as to oppose the change in the magnetic field that produced them.

a) When the bar magnet is moved to the left, the number of lines of magnetic flux through the coil, which are directed from left to right in the figure, decreases. To counteract this change, the coil must set up a field directed to the right. Thus the direction of the induced current in the resistor is to the right.

b) Initially there's no magnetic flux through the coil. When the switch is closed, magnetic flux lines directed to the left are produced. To counteract this change, the current in the resistor will flow from the rear to the front of the resistor.

c) The magnetic field inside the loop produced by the current is directed into the page. As the current decreases rapidly to zero, the magnetic flux through the loop will decrease. To counteract this change, current in the clockwise direction (to the right through \( R \)) will be set up in the circuit.

d) Imagine the bar as part of a rectangular circuit loop on the page whose size increases as the bar moves. The top of the bar becoming positively charged corresponds to a counterclockwise current induced in the loop which in turn induces a magnetic flux out of the page. By Lenz’s law this must be opposite to an increasing flux into the page. Therefore the magnetic field must point into the page.

5. Serway 31.34

For the situation described in Figure P31.33, the magnetic field changes with time according to \( B = (2t^3 - 4t^2 + 0.8) \, \text{T}, \) and \( r_2 = 2R = 5 \, \text{cm}. \) (a) Calculate the magnitude and direction of the force exerted on an electron located at point \( P_2 \) when \( t = 2 \, \text{sec}. \) (b) At what time is this force equal to zero?

SOLUTION:

a) First, let’s construct a circular path concentric to the circular cross-section of the field and that passes through \( P_2. \) The magnetic flux \( \Phi_B \) through the surface bounded by this path is then:
\[ \Phi_B = \oint \mathbf{B} \cdot d\mathbf{a} = B(\pi R^2) = \pi R^2(2t^3 - 4t^2 + 0.8). \]

Faraday’s Law of induction states that
\[ -\frac{d\Phi_B}{dt} = -\pi R^2 (6t^2 - 8t) = \oint E \cdot d\mathbf{s} = E(2\pi r_2) = E(4\pi R), \]

or,
\[ |E| = \frac{R(3t^2 - 4t)}{2}. \]

So the magnitude of the force on an electron at \( t = 2 \, \text{sec} \) and for \( r_2 = 2R = 5 \, \text{cm} \) is:
\[ F = |e|E = \frac{|e|R}{2} (3t^2 - 4t) = 8 \times 10^{-21} \, \text{N}. \]
where |e| = 1.6 \times 10^{-19} \text{ C}. Since the magnetic flux is increasing at t=2, \((d\Phi_B/dt \geq 0)\) the electric field produced at P_2 must be counterclockwise to counteract the change in accordance with Lenz’s Law. Thus the direction of the force on the electron is clockwise.

b) Set

\[ F = \left|e\right| R \left(3t^2 - 4t\right) = 0. \]

So, at \( t = 0 \) and \( t = 4/3 \text{ sec} \), the force will be equal to zero.

6. Serway 31.66

A horizontal wire is free to slide on the vertical rails of a conducting frame, as in Figure P31.66 of your text. The wire has mass \( m \) and length \( \ell \), and the resistance of the circuit is \( R \). If a uniform magnetic field is directed perpendicularly to the frame, what is the terminal speed of the wire as it falls under the influence of gravity?

**SOLUTION:** As the rail falls down under the influence of gravity, the change in the magnetic flux through the circuit is

\[ \frac{d\Phi_B}{dt} = B \ell \frac{du}{dt} = B v. \]

So, the induced EMF in the circuit is

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -Bv \]

and the induced current is then

\[ I = \frac{|\mathcal{E}|}{R} = \frac{Bv}{R} \]

in the counterclockwise direction. This current in turn will give rise to a magnetic force on the sliding wire whose magnitude is

\[ F_B = I \vec{L} \times \vec{B} = I \ell \vec{B} = \frac{\ell^2 B^2 v}{R} \]

and having an upward direction. When this magnetic force becomes equal to the gravitational force, the wire will reach its terminal velocity:

\[ \frac{\ell^2 B^2 v}{R} = mg \]

\[ v = \frac{mgR}{\ell^2 B^2}. \]

7. Serway 32.78

The toroid in Figure P32.78 of your text consists of \( N \) turns and has a rectangular cross-section. Its inner and outer radii are \( a \) and \( b \), respectively. (a) Show that

\[ L = \frac{\mu_0 N^2 h}{2\pi} \ln \left( \frac{b}{a} \right). \] (7.1)

(b) Using this result, compute the self-inductance of a 500-turn toroid for which \( a = 10 \text{ cm} \), \( b = 12 \text{ cm} \), and \( h = 1 \text{ cm} \). (c) In Problem 32.14 of your text, an approximate formula for the inductance of a toroid with \( R \gg r \) was derived. To get a feel for the accuracy of this result, use the expression in that problem to compute the (approximate) inductance of the toroid described here in part (b).

**SOLUTION:**

a) As shown in example 30.5, the magnitude of the field created by a toroid is given by

\[ B = \frac{\mu_0 NI}{2\pi r}. \]

So, the magnetic flux through the toroid is

\[ \Phi_B = \oint \vec{B} \cdot d\vec{a} = \int_a^b B(hdr) = \int_a^b \frac{\mu_0 NH h}{2\pi r} dr = \frac{\mu_0 NH h}{2\pi} \ln \left( \frac{b}{a} \right). \]
Since the inductance of the coil is given by \( L = N\phi_B/I \), therefore

\[
L = \frac{\mu_0 N^2 h}{2\pi} \ln \left( \frac{b}{a} \right).
\]

b) For \( N = 500 \) turns, \( a = 10 \) cm, \( b = 12 \) cm, and \( h = 1 \) cm:

\[
L = \frac{\mu_0 N^2 h}{2\pi} \ln \left( \frac{b}{a} \right) = \frac{(4\pi \times 10^{-7} \text{ Tm/A})(500)^2 (0.01 \text{ m})}{2\pi} \ln \left( \frac{12}{10} \right)
\]

\[= 9.12 \times 10^{-5} \text{ Henry}.\]

c) Using the approximate formula derived in problem 32.14,

\[
L \approx \frac{\mu_0 N^2 A}{2\pi} = \frac{\mu_0 N^2 h(b-a)}{2\pi R} = 9.09 \times 10^{-5} \text{ Henry}
\]

where \( R = 11 \) cm is used. Comparing this result with the one found in (b), we see that this approximate formula gives a reasonably accurate answer.

8. Practice Problems – Don’t Hand In

Chapter 31: 11A, 19, 20, 22, 32, 33.