1. Reading Assignment
Serway Sections 34.1–4.

2. Serway 34.4
Show that \( E = f(x - ct) + g(x + ct) \) satisfies the wave equation (Eq. 34.8 in your text), where \( f \) and \( g \) are any functions.

SOLUTION: The wave equation is given by \( \frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \).
Take \( E = f(x - ct) + g(x + ct) \). Then
\[
\frac{\partial^2 E}{\partial x^2} = f''(x - ct) + g''(x + ct),
\]
where a prime denotes the derivative with respect to the function’s argument. When we take the derivative with respect to time we get a factor of \(-c\) or \(+c\) (for \( f \) and \( g \) respectively). Hence
\[
\frac{\partial E}{\partial t} = -cf'(x - ct) + cg'(x + ct),
\]
\[
\frac{\partial^2 E}{\partial t^2} = c^2 \left( f''(x - ct) + g''(x + ct) \right).
\]
Since \( \mu_0 \epsilon_0 = 1/c^2 \) we have
\[
\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}.
\]

3. Similar to Serway 34.8
Write down expressions for the electric and magnetic fields of a sinusoidal plane electromagnetic wave having a frequency of 3 GHz and traveling in the negative \( z \) direction. The amplitude of the electric field is 300 V/m.

SOLUTION: The electric and magnetic fields are always perpendicular to the direction of the travelling wave in such a way that \( \vec{E} \times \vec{B} \) is the direction of the travelling wave. Thus we can choose \( \vec{E} = E\hat{j} \) and \( \vec{B} = B\hat{i} \) to have the plane wave going in the negative \( z \) direction. Here
\[
E = E_{\text{max}} \cos(kz + \omega t),
\]
\[
B = B_{\text{max}} \cos(kz + \omega t),
\]
where \( k = \omega/c, \omega = 2\pi f \) and \( B_{\text{max}} = E_{\text{max}}/c \). Putting all this together with \( f = 3\text{GHz} \), and \( E_{\text{max}} = 300\text{V/m} \) we have
\[
\vec{E} = 300 \cos(62.8z + 1.88 \times 10^{10}t) \hat{j} \quad \text{V/m},
\]
\[
\vec{B} = 10^{-6} \cos(62.8z + 1.88 \times 10^{10}t) \hat{i} \quad \text{T},
\]
for \( z \) in meters and \( t \) in seconds. Note: An equally valid choice is \( \vec{E} \) in the \( \hat{i} \) and \( \vec{B} \) in the \( -\hat{j} \) direction.

4. Serway 34.20
A monochromatic light source emits 100 W of electromagnetic power uniformly in all directions. (a) Calculate the average electric-field energy density 1 m from the source. (b) Calculate the average magnetic-field energy density at the same distance from the source. (c) Find the wave intensity at this location.

SOLUTION: The intensity of the wave at distance \( r \) is given by \( I = \text{Power}/(4\pi r^2) \). Also the intensity is given by \( I = cu_{\text{avg}} \) where \( u_{\text{avg}} \) is the average energy density. Hence
\[
u_{\text{avg}} = \frac{\text{Power}}{4\pi r^2 c} = 2.65 \times 10^{-8} \quad \text{J/m}^3.
\]
a) and b) The average electric and magnetic field energy densities are equal and add up to \( u_{\text{avg}} \). Hence they are both equal to \( u_{\text{avg}}/2 = 1.32 \times 10^{-8} \text{J/m}^3 \).
c) As already noted \( I = \text{Power}/(4\pi r^2) = 7.96\text{W/m}^2 \).
5. Serway 34.30A
Lasers have been used to suspend spherical glass beads in the Earth’s gravitational field. (a) If a bead has a mass \( m \) and a density \( \rho \), determine the radiation intensity needed to support the bead. (b) If the beam has a radius \( r \), what is the power required for this laser?

**SOLUTION:**

a) We assume the glass bead is perfectly reflecting. The pressure \( P \) is given by \( P = 2I/c \), where \( I \) is the radiation intensity. The force exerted by the radiation must counteract \( mg \), the weight of the bead. Hence \( PA = mg \), where \( A = \pi r^2 \) is the cross-section of the bead, and \( r \) is the radius. We can find \( r \) from \( m = \rho \frac{4}{3} \pi r^3 \), so \( r = \left( \frac{3m}{4\pi \rho} \right)^{1/3} \). Putting this all together,

\[
I = \frac{Pc}{2} = \frac{cmg}{2\pi r^2} = \frac{cmg}{2\pi} \left( \frac{4\pi \rho}{3m} \right)^{1/3} \frac{2}{9\pi} cgm^{1/3} \rho^{2/3}.
\]

b) If the beam has a radius \( R \) (this is not necessarily the same as \( r \) in part a), then the power needed to produce an intensity \( I \) is

\[
\text{Power} = I \cdot \text{area} = I\pi R^2 = \left( \frac{2\pi^3}{9} \right)^{1/3} R^2 cgm^{1/3} \rho^{2/3}.
\]

6. Serway 34.46
A microwave source produces pulses of 20 GHz radiation, with each pulse lasting 1 ns. A parabolic reflector (\( R = 6 \text{ cm} \)) is used to focus these into a parallel beam of radiation (ie, plane waves), as in Figure P34.46 of your text. The average power during each pulse is 25 kW. (a) What is the wavelength of these microwaves? (b) What is the total energy contained in each pulse? (c) Compute the average energy density inside each pulse. (d) Determine the amplitude of the electric and magnetic fields in these microwaves. (e) If this pulsed beam strikes an absorbing surface, compute the force exerted on the surface during the 1 ns duration of each pulse.

**SOLUTION:**

a) The wavelength is \( \lambda = c/f = 1.5 \times 10^{-2} \text{ m} \).

b) The energy contained in a pulse is \( \Delta U = \text{Power} \cdot T = 25 \times 10^{-6} \text{ J} \).

c) We can find the intensity \( I \) from \( I = \text{Power}/\pi R^2 \), and then the average energy density is \( u_{\text{avg}} = I/c = \text{Power}/\pi R^2 c = 7.4 \times 10^{-3} \text{ J/m}^3 \) during the pulse.

d) \( u_{\text{avg}} = \epsilon_0 E_{\text{max}}^2/2 = B_{\text{max}}^2/(2\mu_0) \), giving \( E_{\text{max}} = 4.1 \times 10^4 \text{ V/m} \), and \( B_{\text{max}} = 1.4 \times 10^{-4} \text{ T} \).

e) The beam has a constant cross-section \( \pi R^2 \). The pressure on an absorbing surface (no reflection) is \( P = I/c = \text{Power}/\pi R^2 c \), hence the force is \( F = P \pi R^2 = \text{Power}/c = 8.3 \times 10^{-5} \text{ N} \), during the pulse.

7. Serway 34.52
A possible means of space flight is to place a perfectly reflecting aluminized sheet into Earth’s orbit and use the light from the Sun to push this solar sail. Suppose a sail of area \( 6 \times 10^4 \text{ m}^2 \) and mass 6000 kg is placed in orbit facing the Sun. (a) What force is exerted on the sail? (b) What is the sail’s acceleration? (c) How long does it take the sail to reach the Moon, \( 3.84 \times 10^8 \text{ m} \) away? Ignore all gravitational effects, assume that the acceleration calculated in part (b) remains constant, and assume a solar intensity of 1380 W/m².

**SOLUTION:**

a) The force exerted on the sail is \( F = \text{pressure} \times \text{area} = 21A/c = 0.55 \text{ N} \).

b) The acceleration is \( a = F/m = 9.2 \times 10^{-5} \text{ m/s}^2 \).

c) The time it takes to reach the moon is \( t = \sqrt{2d/a} = 2.9 \times 10^6 \text{ s} \approx 34 \text{ days} \).

8. Practice Problems – Don’t Hand In

In Serway Chapt. 34: 9, 11, 13, 23, 27, 45, 53.