3-3

(a) \( y(x, t) = x^2 + v^2 t^2 \)

\[
\frac{\partial y}{\partial x} = 2x, \quad \frac{\partial^2 y}{\partial x^2} = 2
\]

\[
\frac{\partial y}{\partial t} = 2vt, \quad \frac{\partial^2 y}{\partial t^2} = 2v^2
\]

\[
\frac{\partial^2 y}{\partial t^2} = 2v^2 = v^2 \frac{\partial^2 y}{\partial x^2}
\]

(b) \( x^2 + v^2 t^2 = \frac{1}{2} (x + vt)^2 + \frac{1}{2} (x - vt)^2 \)

(c) \( y(x, t) = \sin(x) \cos(vt) \)

\[
\frac{\partial y}{\partial x} = \cos(x) \cos(vt), \quad \frac{\partial^2 y}{\partial x^2} = -\sin(x) \cos(vt)
\]

\[
\frac{\partial y}{\partial t} = \sin(x) (-v) \sin(vt), \quad \frac{\partial^2 y}{\partial t^2} = \sin(x) (-v^2) \cos(vt)
\]

\[
\frac{\partial^2 y}{\partial x^2} = -\sin(x) \cos(vt) = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}
\]

Using the ANGLE ADITION FORMULA

\[ \sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B) \]

\[
(\text{x}) \quad \sin(x + vt) = \sin(x) \cos(vt) + \cos(x) \sin(vt)
\]

\[
(\text{y}) \quad \sin(x - vt) = \sin(x) \cos(vt) - \cos(x) \sin(-vt)
\]

But \( \cos(-vt) = \cos(t) \) EVEN

\( \sin(-vt) = -\sin(vt) \) ODD

Adding \( \text{x} \) and \( \text{y} \), and dividing by two

\[
\gamma(x, t) = \sin(x) \cos(vt) = \frac{1}{2} \sin(x + vt) + \frac{1}{2} \sin(x - vt)
\]