Single-Slit Diffraction

**Demo: Single-Slit Diffraction**

Light, Wavelength $\lambda$

Diffracting through Slit Width $a$

- Divide slit into $N$ chunks, size $\Delta x = a/N$

- Phase difference $\Delta \phi$ between neighbor chunks

$$\Delta \phi = 2\pi \frac{\Delta x \sin \theta}{\lambda}$$

- $E(\theta) = \sum_{n=1}^{N} \left( \text{field of chunk } n \text{ at point } P \right)$

$$= \sum_{n=1}^{N} (\Delta E_0) \sin \left( 2\pi ft + n \Delta \phi \right)$$
Phasor diagram = Polygon of chunk contributions

\[ \Delta \phi = 2\pi \frac{\Delta x \sin \theta}{\lambda} \]

gives the biggest intensity?

A

\[ \Delta \phi = 0 \]

At center, \( \Theta = \Delta \phi = 0 \)

\[ E_{\text{center}} = N \Delta E_0 \sin(2\pi f t) \]

\[ I_{\text{center}} = \frac{\varepsilon_0}{2} \left( \frac{E_{\text{center}}}{2} \right)^2 = \frac{\varepsilon_0}{2} N^2 (\Delta E_0)^2 \]

Q: At what \( \Delta \phi \) will \( E(\theta) = 0 \)?

A

Polygon closes when \( N \Delta \phi = 2\pi \)

\[ N \Delta \phi = 2\pi \quad N \left( 2\pi \frac{\Delta x \sin \theta}{\lambda} \right) = 2\pi \]

\[ (N \Delta x) \sin \theta = \lambda \]

\[ a \sin \theta = \lambda \quad \text{First dark spot single slit} \]

\[ a \sin \theta = \lambda \quad \text{First light spot double slit} \]

When path length difference between top & bottom is one wiggle, all phases are added

\[ \Rightarrow \text{complete cancellation!} \]
General Case:

- Chunk contributions form arc of a circle radius $R$, angle $2\alpha$.

Q: What is $E(\theta)$, in terms of $R$ and $\alpha$?
A: $E(\theta)/2 = R \sin \alpha$

$$E(\theta) = 2R \sin \alpha$$

Need $R$ and $\alpha$.

$$2\alpha = \text{[Angle phasor rotates]} = N \Delta \phi$$

$$\alpha = \frac{N}{2} \Delta \phi = \frac{N}{2} \left( 2\pi \frac{\Delta x \sin \theta}{\lambda} \right) = \pi \left( N \Delta x \right) \frac{3\sin \theta}{\lambda} = \pi a \sin \theta$$

Arc length = $R \cdot (\text{angle of arc})$

$$E_{\text{center}} = N \Delta E_0 = R \cdot 2\alpha$$

$$R = \frac{E_{\text{center}} \left( \frac{1}{2\alpha} \right)}{2\alpha}$$

$$E(\theta) = 2R \sin \alpha = 2 \frac{E_{\text{center}}}{\sin \frac{\alpha}{\lambda}}$$

$$I_{av} = \frac{1}{\text{center}} \frac{\sin^2 \frac{\alpha}{\lambda}}{\alpha^2}$$

Computer DEMO Dugan's Dynamic Phasors
**BEATS, A.M. RADIO, and FOURIER TRANSFORMS**

**DEMO: Beats**

Adding two waves of the same amplitude, different frequencies,

\[ A(t) = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) \]

Envelope (smooth curve over wiggles)

Two beats per period of envelope.

When \( f_1 \) and \( f_2 \) are close, get beats.

Phasor Diagram

\[ A(t) = A_e(t) \sin(\phi(t)) \]

\( f_1 \approx f_2 \Rightarrow \) nearly rigid \( \Rightarrow \) \( A_e \) changes slowly \( \Rightarrow \) envelope
\[ A(t) = A_1 \sin (2\pi f_1 t) + A_2 \sin (2\pi f_2 t) \]

- Looks like a wave, frequency \( \bar{f} = \frac{f_1 + f_2}{2} \), pulsing on and off.

- Write \( f_1 = \bar{f} - \frac{\Delta f}{2} \), \( f_2 = \bar{f} + \frac{\Delta f}{2} \), \( \Delta f = f_1 - f_2 \)

\[ A(t) = A_1 \sin(2\pi \left( \bar{f} - \frac{\Delta f}{2} \right) t) + A_2 \sin(2\pi \left( \bar{f} + \frac{\Delta f}{2} \right) t) \]

\[ = A_1 \sin(2\pi \bar{f} t - \pi \Delta ft) + A_2 \sin(2\pi \bar{f} t + \pi \Delta ft) \]

Now, \( \sin (B+C) = \sin B \cos C + \cos B \sin C \)
\( \sin (B-C) = \sin B \cos C - \cos B \sin C \)

\[ A(t) = A_1 \left[ \sin(B-C) + \sin(B+C) \right] \]

\[ = A_1 \left[ \sin B \cos C + \cos B \sin C + \sin B \cos C - \cos B \sin C \right] \]

\[ = 2A_1 \sin B \cos C = 2A_1 \sin(2\pi \bar{f} t) \cos(\pi \Delta ft) \]

\[ = \underbrace{2A_1 \cos(\pi \Delta ft)}_{\text{Envelope} A_E(t)} \underbrace{\sin(2\pi \bar{f} t)}_{\text{Carrier Frequency at } \bar{f} = \frac{f_1 + f_2}{2}} \]

Oscillates at \( \Delta f \), but two beats per period.
\[ \Delta f = f_2 - f_1 \]
\[ \frac{\Delta f}{2} = \frac{f_2 + f_1}{2} \]

\[ \phi_E(t) = \text{angle of } A_2 = 2\pi f_1 t + \pi \Delta f t \]
\[ = 2\pi f_1 t + \pi (f_2 - f_1) t = \pi f_1 t + \pi f_2 t = 2\pi f_0 t \]

\[ A_E(t) = 2A_1 \cos (\pi \Delta f t) \quad \text{Envelope} \]

\[ A(t) = A_E(t) \sin \phi_E(t) \]
\[ = \left[ 2A_1 \cos (\pi \Delta f t) \right] \sin (2\pi f_0 t) \]

\[ \text{Envelope} \quad \text{Carrier} \]

\[ \text{WARNING: Two beats per period.} \]

This is what AM radio does!

- AM = Amplitude Modulation

\[ \Delta f = \text{frequency of note } A = 440 \text{ Hz} \]
\[ f = \text{frequency of radio station } 80 = 800 \text{ kHz} \]

Signal is \[ 2A_1 \cos (\pi \Delta f t) \sin (2\pi f_0 t) \]
\[ \text{and } 799,780 \text{ Hz} \]