Phasors are our geometrical method for adding the amplitude of different waves.

\[ E(t) \quad E_0 \quad \text{wt} \quad \frac{Q: \text{What is } E(t) \text{?}}{A: E(t) = E_0 \sin(\omega t)} \]

- The phasor for the wave \( E_0 \sin(\omega t) \) has length \( E_0 \), angle \( \omega t \).
- The vertical component of the phasor is \( E(t) \).

Q: What is the intensity \( I(t) \)?

\[ I(t) = \frac{E_0 E^2}{2} \left( \sin^2 \omega t \right) \quad \text{[EM wave reading]} \]

- The intensity of a wave is proportional to the amplitude squared.
- We won't care much about the constant \( E_0 \) in this part.

Q: What is the average intensity \( I_{av} \) over time?

\[ I = \frac{E_0 E_0^2}{2} \quad \text{average } \sin^2 = \text{average } \cos^2 = \frac{1}{2} \]

- The average intensity of a wave is proportional to the square of the phasor's length.
Phasor with two wave sources

\[ E_2(t) = E_0 \sin(\omega t + \phi) \]

\[ E_1(t) = E_0 \sin(\omega t) \]

\[ E(t) = E_1(t) + E_2(t) = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi) \]

- The relative angle between the two phasors remains at \( \phi \).
- The whole triangle rotates at frequency \( \omega t \).
- \( E_R \), the length of the resultant phasor, stays fixed.

Q: In terms of \( E_R \), \( \alpha \), and \( \omega t \), what is \( E_1(t) + E_2(t) \)?

A: \[ E_1(t) + E_2(t) = E_R \sin(\omega t + \alpha) \]
The vector sum of the phasors has a vertical component: the sum of the waves.

Q: What is $\alpha$?
A: Isosceles $\Rightarrow \alpha = \frac{\phi}{2}$

$x + \phi = 180^\circ$ 
$x + 2\alpha = 180^\circ \Rightarrow 2\alpha = \phi \Rightarrow \alpha = \frac{\phi}{2}$

Q: What's $E_R/2$?
A: $E_R/2 = E_0 \cos(\phi/2)$

$E_R = 2E_0 \cos(\phi/2)$

The sum of the equal amplitude waves $E_1(t) + E_2(t) = E_0 \sin(wt) + E_0 \sin(wt + \phi)$ has magnitude $E_R = 2E_0 \cos \phi/2$

phase $\alpha = \phi/2$

$E(t) = E_R \sin(wt + \alpha) = 2E_0 \cos(\phi/2) \sin(wt + \phi/2)$

The intensity of the sum:

$I_R = \frac{1}{2} (E_0^2) E_R^2 = \left(\frac{E_0^2}{2}\right) \left(4E_0^2 \cos^2(\phi/2)\right)$
The average intensity of just $I_1(t)$ is

$$\bar{I}_1 = \frac{\varepsilon_0}{2} E_0^2$$

Q: Plot $I_R$ and $I_1$ as a function of the phase difference $\phi$, for $0 < \phi < 2\pi$.

Q: What happens for a phase difference $\phi = \pi$ (one half wiggle)? Draw the phasor diagram.
A: Destructive interference. No intensity.

$\phi = \pi$

Q: What happens for a phase difference of zero, or $2\pi$? (No wiggles, two wiggles?) Draw the phasor diagram.
A: Constructive interference. Intensity four times $I_1$.

Constructive interference of two sources gives more than twice the light.