**SCALING**

What happens to the power, energy density, and total energy of a pulse when its size changes?

TRAVELING

Same Shape

Reference Pulse

Shape \( y_1 \)

\[
P_1 = -\tau \frac{\partial y_1}{\partial x} \frac{\partial y_1}{\partial t}
\]

Pulse Amplitude Changed

\( y_2 = Ay_1 \)

\[
P_2 = -\tau \frac{\partial y_2}{\partial x} \frac{\partial y_2}{\partial t}
\]

\[
= -\tau \frac{\partial (Ay_1)}{\partial x} \frac{\partial (Ay_1)}{\partial t}
\]

\[
= A^2 \left( \tau \frac{\partial y_1}{\partial x} \frac{\partial y_1}{\partial t} \right)
\]

\[
= A^2 P_1
\]

\[
u_1(x) = \frac{1}{2} \mu \left[ \frac{\partial y_1}{\partial t} \right]^2 + \frac{1}{2} \tau \left( \frac{\partial y_1}{\partial x} \right)^2
\]

\[
u_2(x) = \frac{1}{2} \mu \left[ \frac{\partial (Ay_1)}{\partial t} \right]^2 + \frac{1}{2} \tau \left( \frac{\partial (Ay_1)}{\partial x} \right)^2
\]

\[
= A^2 \nu_1(x)
\]

\[
\text{(Total Energy)}_1 = \int u_1(x) \, dx
\]

\[
\text{(Total Energy)}_2 = \int A^2 \nu_1(x) \, dx
\]

\[
= A^2 \left( \text{Total Energy} \right)_1
\]

Power, Energy Density, and Total Energy

All Scale with the Square of the Amplitude (Fixed shape)
Reference Pulse
Shape \( y_1(x,t) = f(x-ut) \)

Slope \( \frac{\partial y_1}{\partial x}(x,t) = f'(x-ut) \)

Chunk Velocity \( \frac{\partial y_1}{\partial t}(x,t) = -v f'(x-ut) \)

Pulse Width Changed
\( y_2(x,t) = f(x-ut) \)

Slope \( \frac{\partial y_2}{\partial x}(x,t) = \frac{1}{v} f'(x-ut) \)

Chunk Velocity \( \frac{\partial y_2}{\partial t}(x,t) = -\frac{v}{v} f'(x-ut) \)

\( P_1 = \text{Power} = -v \frac{\partial y_1}{\partial x} \frac{\partial y_1}{\partial t} \)

\( P_2 = \text{Power}_2 = -v \frac{\partial y_2}{\partial x} \frac{\partial y_2}{\partial t} \)

Energy Density \( u_1(x,t) = \frac{1}{2} \rho (\frac{\partial y_1}{\partial x})^2 + \frac{1}{2} \mu (\frac{\partial y_1}{\partial t})^2 \)

\( = \frac{1}{2} \rho \dot{f}^2 + \frac{1}{2} \mu v^2 \dot{f}^2 \)

\( = \frac{1}{2} \left( \frac{1}{(\Delta x)^2} f(x,t) + \frac{1}{\Delta x} f'(x,t) \right) \)

\( = \frac{1}{(\Delta x)^2} u_1 \left( \frac{x}{\Delta x}, t \right) \)

\( \Rightarrow u_2 \text{ at } \Delta x \)

\( \Rightarrow u_1 \text{ at } \frac{x}{\Delta x} \)
\[(Total\ Energy)_1 = \int u_1(x,t)\ dx\]

\[u_1(x)\]

\[u_2(x)\]

\[\text{Down by } \frac{\Delta x}{2}\]

\[\text{Stretched by } \Delta x\]

\[\text{Integral} = \text{Area underneath}\]

\[\frac{1}{\Delta x} = \frac{(\Delta x)^2}{\text{Height}}\]

\[(Power)_2 = \frac{A^2}{\Delta x^2} (Power)_1\]

\[U_2 = \frac{A^2}{\Delta x^2} U_1\]

\[(Total\ Energy)_2 = \frac{A^2}{\Delta x} (Total\ Energy)_1\]

\[\text{Exact if}\]

\[\text{shape of}\]

\[\text{pulse kept}\]

\[\text{fixed}\]

\[\text{Corresponds to Dimensional Analysis}\]

\[P = -\tau \frac{\partial v}{\partial x} \frac{\partial x}{\partial t}\]

\[\partial y \sim A\]

\[\partial t \sim \Delta t = \Delta x/v\]

\[\frac{P + A^2}{\Delta x (\Delta x/6)}\]

\[u = \frac{1}{2} \mu \left(\frac{\partial v}{\partial t}\right)^2 + \frac{1}{2} \tau \left(\frac{\partial v}{\partial x}\right)^2\]

\[\sim \frac{1}{2} \mu \left(\frac{A^2}{\Delta x/6}\right)^2 + \frac{1}{2} \tau \frac{A^2}{(\Delta x)^2}\]

\[\sim \frac{v^2}{2} + \tau \left(\frac{A^2}{\Delta x^2}\right)\]

\[\text{Total Energy} = \int u\ dx \sim u\Delta x \sim \tau \left(\frac{A^2}{\Delta x}\right)\]
Reflection and Transmission at Discontinuities in Strings

Two different strings \( \mu_1 \) & \( \mu_2 \) tied together:

\[ \mu_1 \quad \mu_2 \]

\( \Delta x \) Initial Pulse

**Demo: COMPUTER**

Step Up: Time to Run = 0.1025

* \( \mu_2 > \mu_1 \)
* String continuous at peak
* Reflected pulse inverted
  \( \Delta x \) \( \Rightarrow \) Heavy \( \mu_2 \) like fixed boundary

* Transmitted pulse smaller, narrower \( \Delta x_T < \Delta x_I \)
* Pulses all same shape

Reflection here similar to reflection of light off of water - water "thick string", air "thin string"

Some light transmitted into water
Some reflected \( \Rightarrow \) see face in paddle
Step Down: Time to Run = 0.04125

- $\mu_2 < \mu_1$
- String continuous
- Reflected pulse not inverted (light $\mu_2$ like free boundary)
- Transmitted pulse wider, higher: $\Delta x_T > \Delta x_I$, $A_T > A_I$ not always
- Pulses all same shape

You will verify two laws in Pythag:

\[
A_I + A_R = A_T
\]
Continuity of String

\[
P_I + P_R = P_T
\]
Power into = Power away from knot
Conservation of Energy

In the homework, you'll use SCALING and these two laws to solve for $A_R$ and $A_T$. 