PHYSICS 214

WAVES on a String; Sound, Light $v_s$

OPTICS: Interference, Diffraction; Fibers, Films, & Slits $v_e$

& PARTICLES: Quantum Mechanics $\frac{1}{2}$

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PENDULUM DEMO: oscillate, spin, damping [ignored]

Hold at $\theta_0 \approx 30^\circ > 0$; drop. Catch after 2 periods.

Exercise 1: Draw $\theta(t)$

Exercise 2: Mark the period of the motion $T$ on your graph. In terms of $T$, what is the frequency $f$ in cycles per second?

Exercise 3: During which time is the angular acceleration $\theta'' > 0$? When $\theta'' < 0$, Restoring Force
The Equation of Motion for the Pendulum

Physics 112, Serway reading assignment

1. Free Body Diagram
2. Sum forces: set to 0 ma
3. Find differential equation satisfied by \( \theta(t) \)

**EQUATION OF MOTION**

\[
\frac{d^2 \theta}{dt^2} = -\frac{g}{L} \sin \theta
\]

The equation of motion tells you how the velocity changes in the future, given the positions and velocities now.

We can't write down the exact solution to this differential equation. We need to find a numerical solution.

The solution is \( \theta(t) \) which solves the equation of motion.
We can't store $\theta(t)$ on the computer:

\[ [8 \text{ bytes per } \theta] \times [\infty \text{ different times}] = \infty \text{ Gbyte hard disk} \]

We approximate $\theta(t)$ by a set of equally spaced points $\theta(t_n)$ where $t_n = n \delta t$

Our numerical equation of motion will tell us $\theta(t_{n+1})$ given $\theta(t_n)$ and $\theta(t_{n-1})$.

Future \( \leftarrow \) (Present \& past)

We need a way to calculate $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ approximately, given $\theta(t_n)$. 
**FIRST DERIVATIVE**

By definition \( \frac{d\Theta}{dt} = \lim_{\varepsilon \to 0} \frac{\Theta(t+\varepsilon) - \Theta(t)}{\varepsilon} = \frac{\text{Rise}}{\text{Run}} \)

Approximately \( \frac{d\Theta}{dt} \approx \frac{\Theta(t_n + st) - \Theta(t_n)}{st} = \frac{\Theta_{n+1} - \Theta_n}{st} \)

Q: At what time is this \( \frac{d\Theta}{dt} \)? \( t_n \)? \( t_{n+1} \)?
A: At the midpoint, \( t_n + \frac{st}{2} \).

Similarly, \( \frac{d\Theta}{dt} \approx \frac{\Theta_n - \Theta_{n-1}}{st} = \frac{\text{Rise}}{\text{Run}} \)
SECOND DERIVATIVE

By definition \[ \frac{d^2 \theta}{dt^2} = \frac{d}{dt} \left( \frac{d \theta}{dt} \right) \]

Approximately \[ \frac{d^2 \theta}{dt^2} \approx \frac{\frac{d \theta}{dt}(t + \frac{\delta t}{2}) - \frac{d \theta}{dt}(t - \frac{\delta t}{2})}{\delta t} \]

\[ \frac{d^2 \theta}{dt^2} = \frac{(\theta_{n+1} - \theta_n)}{\delta t} - \frac{(\theta_n - \theta_{n-1})}{\delta t} \]

\[ \frac{d^2 \theta}{dt^2} = \frac{(\theta_{n+1} - \theta_n) - (\theta_n - \theta_{n-1})}{\delta t^2} \]

\[ \frac{d^2 \theta}{dt^2} = \frac{\theta_{n+1} - 2\theta_n + \theta_{n-1}}{\delta t^2} \]

We'll use this again

Our numerical equation of motion is

\[ \frac{\theta_{n+1} - 2\theta_n + \theta_{n-1}}{\delta t^2} = \frac{d^2 \theta}{dt^2} = -\frac{g}{L} \sin \theta \]

\[ \theta_{n+1} = (\delta t)^2 \left( \frac{g}{L} \sin \theta \right) + 2\theta_n - \theta_{n-1} \]

Future ↔ Present and Past

Homework Set 2