1. Air Wedge

An air wedge is formed between two very thick glass plates separated at one edge by a very fine wire. When the wedge is illuminated from above by light with a wavelength of 600 nm, 30 dark fringes are observed. Calculate the radius of the wire. (Consider only reflections from the top and bottom surfaces of the wedge.)

**Solution:** Waves reflected from the bottom surface of the air wedge are phase shifted by half a wavelength, while those reflected from the top surface are not shifted. Thus a dark fringe will appear wherever the wedge thickness is a multiple of \( \frac{\lambda}{2} \), where \( \lambda \) is the wavelength of light in air. Since there are 30 fringes, these occur at thicknesses of \( 0, \frac{\lambda}{2}, \frac{2\lambda}{2} \ldots 29\frac{\lambda}{2} \). Therefore the radius of the wire is roughly \( 29\lambda/4 = 4.35 \mu \text{m} \).

2. Thin Oil Film

A film of oil \( (n = 1.4) \) sits on a flat glass plate \( (n = 1.6) \). When white light (coming from air) strikes the film vertically, the colors most enhanced in the reflected beam have \( \lambda_1 = 420 \text{ nm} \) and \( \lambda_2 = 630 \text{ nm} \). Which of the following is closest to being the minimum thickness of the film?

- (A) 150 nm
- (B) 210 nm
- (C) 225 nm
- (D) 315 nm
- (E) 450 nm
- (F) 485 nm

**Solution:** There is a \( \pi \) phase change for the reflection off of the top of the film but not the bottom. Therefore there will be destructive interference when \( 2nt = m\lambda \), where \( t \) is the film thickness. We get destructive interference for 450 nm and 600 nm light with no missing wavelengths in between so therefore, \( m(450) = (m-1)(600) \) which means that \( m = 4 \). The thickness of the film is then \( t = m\lambda/(2n) = 4 \times (450/8) = 675 \text{ nm} \).

b) At the top the film is very thin. A band across the top of the film appears dark for all wavelengths in the visible range. What is the maximum thickness of the film in this band?

**Solution:** The smallest value of \( t \) for which the interference is constructive is given by \( 2nt = \lambda/2 \). For smaller \( t \), we tend to have partial destructive interference, with complete destructive interference at \( t = 0 \). So for a dark band, we need \( t \ll \lambda/(4n) \) for all wavelengths in the visible range (\( \lambda \) between 400 and 700 nm). We conclude that the thickness \( t \) of the film must be much less than 75 nm at the top of the film.

3. Thin Film

A thin soap film \( (n = 4/3) \) is suspended vertically in air. The film is viewed by reflected light. The light reflected from the film at a certain point is missing the wavelengths 450 nm and 600 nm, with no missing wavelengths between the two.

a) What is the thickness of the soap film at this point?

**Solution:** The enhanced colors have constructive interference between the reflection off the film and the reflection off the glass. For constructive interference, the phase shift

\[
\phi = \frac{2\pi}{\lambda_{oil}2t}
\]

\( \phi \) must equal 0, 2\( \pi \), 4\( \pi \), ..., where \( t \) is the plate thickness, \( \lambda_{oil} = \lambda/n_{oil} \) is the wavelength of light in oil, and \( \lambda \) is the wavelength of light in air. This gives the condition that \( t = m\lambda/(2n_{oil}) \) where \( m \) is an integer. For the two wavelengths given, we get \( t = 225 \text{ nm} \) and \( t = 150 \text{ nm} \) and we see that \( t = 450 \text{ nm} \) is the minimum thickness that satisfies both conditions. (E) is the correct answer.

b) At the top the film is very thin. A band across the top of the film appears dark for all wavelengths in the visible range. What is the maximum thickness of the film in this band?

**Solution:** The smallest value of \( t \) for which the interference is constructive is given by \( 2nt = \lambda/2 \). For smaller \( t \), we tend to have partial destructive interference, with complete destructive interference at \( t = 0 \). So for a dark band, we need \( t \ll \lambda/(4n) \) for all wavelengths in the visible range (\( \lambda \) between 400 and 700 nm). We conclude that the thickness \( t \) of the film must be much less than 75 nm at the top of the film.

4. Double Slit Diffraction

You are given a double slit illuminated by coherent light where the slit
separation is \(d\) and the slit width is \(a\). If \(d = 7a\), how many bright fringes will appear within the central diffraction maximum? Explain your answer.

(A) 7  
(B) 8  
(C) 12  
(D) 13  
(E) 14  
(F) 15

**SOLUTION:** Where the seventh interference fringe would be, you will get the first diffraction minimum. Therefore, you will see a total of 13 fringes in the central maximum corresponding to \(m = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6\). The correct answer is (D).

5. Single Slit Diffraction

Consider the diffraction pattern of a single slit of width \(a\), as observed on a screen a distance \(L\) from the slit. Find an approximate formula relating the peak intensity of the central maximum, \(I_{\text{max}}\), to the intensity of the light at the slit, \(I_{\text{slit}}\), the width of the slit \(a\), the wavelength \(\lambda\), and the distance \(L\) from the slit to the screen. You may assume that \(\frac{a}{L}\) is much less than 1.

**SOLUTION:** Let \(l\)=the length of the slit. Then if \(P_{\text{slit}}\) is the power passing through the slit, from the relation between intensity and power we have

\[P_{\text{slit}} = I_{\text{slit}}aL\]

Let \(P_{\text{screen}}\) = the power in the central diffraction peak, at the screen. Then we have (approximately)

\[P_{\text{screen}} \approx I_{\text{max}} \frac{\lambda}{a} L\]

If energy is conserved, all the power which passes through the slit ends up on the screen; let us assume that most of it falls within the central diffraction peak. Then \(P_{\text{screen}} \approx P_{\text{slit}}\), so

\[I_{\text{max}} \frac{\lambda}{a} L \approx I_{\text{slit}}aL\]

This is the required formula.

6. Sound Diffraction

A large speaker is aimed at the center of a wall 100 m away. The speaker opening is rectangular and .3 m wide by .9 m high. It emits sound with frequency 3430 Hz. How far along the wall, as measured from its center, should someone stand if she doesn’t wish to hear the sound? Assume a sound speed of 343 m/s. You may also assume that the angles in this problem are small.

(A) 16.5 m  
(B) 11 m  
(C) 33 m  
(D) 3 m  
(E) 1.6 m  
(F) 300 m

**SOLUTION:** The diffraction minimum occurs when \(\theta \approx \lambda/d = v/(df) = .33\). Since the wall is 100 m from the speaker, she should stand 33 m from the wall’s center. (C) is the correct answer.

7. Resolution

Suppose that we have a laser with \(\lambda = 623 \text{ nm}\), emitting a beam that is 2 mm in diameter, and that is collimated to the diffraction limit. How big a spot would be produced on the surface of the moon 376 \(\times 10^3\) km away from such a device?

**SOLUTION:** For a laser of wavelength \(\lambda\) that is 2 mm in diameter, the angular spread of the beam due to diffraction is

\[
\sin \theta_{\text{min}} \approx 1.22 \frac{\lambda}{d} \approx \theta_{\text{min}} \approx r_{\text{moon}}/d_{\text{moon}}
\]

where \(r_{\text{moon}}\) is the radius of the spot on the moon, and \(d_{\text{moon}}\) is the distance to the moon, so we get, putting in the numbers, that \(r_{\text{moon}} \approx 143\) km.