Physics 214 Spring 99—Problem Set 7—Optional Problems

Handout March 9, 1999

1. Cameras

The picture size on ordinary 35-mm camera film is 24 × 36 mm. Focal lengths of lenses available to 35-mm cameras typically include 28 mm, 35 mm, 50 mm (the “standard” lens), 85 mm, 100 mm, 135 mm, 200 mm, and 300 mm, among others. Which of these lenses should be used to photograph the following objects, assuming the object is to fill as much of the picture area as possible?

a) A cathedral 100 m high and 150 m long, at a distance of 150 m.

SOLUTION: In a camera lens, the object is very far away (essentially at infinity) and so the image is located at the focal point. The magnification is then $M = \frac{h}{q} = -\frac{f}{p}$. To fit the cathedral image on the film, we want the (de-)magnified image to be less than 36 mm in length: hence we want $h' = |M|h \leq 36$ mm. With an object distance of $p=150$ m, we have

$$\frac{f}{150 \text{ m}} 150 \text{ m} \leq 36 \text{ mm}$$

which gives $f \leq 36$ mm. So we would use a 35 mm (wide-angle) lens.

b) An eagle with a wingspan 2.0 m, at a distance of 15 m.

SOLUTION:

Following the same reasoning as in part (a), we have $p=15$ m, so we want

$$\frac{f}{15 \text{ m}} 2 \text{ m} \leq 36 \text{ mm}$$

which gives $f \leq 270$ mm. So we would use a 200 mm (telephoto) lens.

2. Optics in a glass rod

Both ends of a glass rod 1 cm in diameter, of index 1.50, are ground and polished to convex hemispherical surfaces of radius 5 cm at the left end and radius 10 cm at the right end. The length of the rod between vertices is 60 cm. An arrow 1 mm long, at right angles to the axis and 20 cm to the left of the first vertex, constitutes the object for the first surface. (See below)

a) What constitutes the object for the second surface?

SOLUTION: The image formed by the first surface will be the object for the second surface. To find this image, we use the relation for a refracting surface

$$\frac{n}{p} + \frac{n'}{q} = \frac{n' - n}{R}$$

In this case, $n=1$, $n'=1.5$, $p=20$ cm, $R=5$ cm, and we want to find $q$. Plugging in and solving gives $q=30$ cm. Since $q$ is positive, the image is formed to the right of the first surface. The magnification is $M = -\frac{h}{p} = -1.5$, so the image is inverted.

b) What is the object distance for the second surface?

SOLUTION: The distance from the image to the second surface is $60 - 30 = 30$ cm; this is the object distance for the second surface.

c) Is the object real or virtual?
SOLUTION: Since the object is located in front of the second surface, it is a real object.

d) What is the position of the image formed by the second surface?

SOLUTION: We again use the equation
\[
\frac{n}{p} + \frac{n'}{q} = \frac{n'-n}{R}
\]
In this case, \(n'=1\), \(n=1.5\), \(p=30\) cm, \(R = -10\) cm, and we want to find \(q\). Plugging in and solving gives \(\frac{q}{q}=0\), which implies that the image is at infinity.

3. Serway, Chapter 36, pg. 1085, Problem 14

SOLUTION: (a). Since the image is inverted, we know that the magnification \(M = -q/p = -4\), so \(q = 4p\) is positive. The image is in front of the mirror. We also know that \(p - q = 60\) cm. Substituting, we have \(4p - p = 3p = 60\) cm, so \(p = 20\) cm. Then \(q = 80\) cm, and using the mirror equation
\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]
we find that \(f = 16\) cm.

(b). For a convex mirror, the image is always virtual, so \(q\) is negative. Thus, the magnification is \(M = -q/p = 0.5\), so \(q = -0.5p\). We also know that \(p - q = 20\) cm. (Remember that \(q\) is negative; the image is behind the mirror). Substituting, we have \(p + 0.5p = 1.5p = 30\) cm, so \(p = 13.33\) cm. Then \(q = -6.67\) cm, and using the mirror equation
\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]
we find that \(f = -13.33\) cm. Since \(f = R/2\), we have \(R = -26.67\) cm.

4. Serway, Chapter 36, pg. 1088, Problem 55

SOLUTION: (a). To find the index of refraction, we use the lensmaker’s equation:
\[
(n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}
\]
We are given \(R_1 = 9\) cm and \(R_2 = -11\) cm. The focal length is \(f = 5\) cm. From the equation above
\[
n = 1 + \frac{1}{5} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]
which works out to \(n=1.99\)

(b).

Let us first describe qualitatively what will happen. Light from the object entering the lens will form an image behind the lens; let this be “image 1”. This image acts as an object for the mirror; if the image is in front of the mirror, it is a real object. The mirror forms an image of this object; let us call this “image 2”. This image, which turns out to be formed between the mirror and the lens, forms a real object for the lens. It is a real object because, in this case, the light reflected from the mirror is passing through the lens from right to left; objects to the right of the lens are thus real objects. The image of this object formed by the lens is the “final image”. This is illustrated below.

![Diagram](image-url)

We can find the position and size of this final image by just applying the lens and mirrors equations in series. We start with the image 1, formed by the lens. The object distance is \(p_1 = 8\) cm; the focal length of the lens
is \( f_{\text{lens}} = 5 \text{ cm} \); so, using the lens equation
\[
\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_{\text{lens}}}
\]
gives \( q_1 = 13.33 \text{ cm} \). Image 1 is formed 13.33 cm to the right of the lens. The magnification is \( M_1 = -q_1/p_1 = -13.33/8 = -1.67 \). The image is inverted.

This image acts as a real object for the mirror. Its object distance for the mirror is \( p_2 = 20 \text{ cm} - q_1 = 6.67 \text{ cm} \). Using the mirror equation
\[
\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_{\text{mirror}}}
\]
where the mirror focal length is \( f_{\text{mirror}} = R/2 = 4 \text{ cm} \). Solving for \( q_2 \), we get \( q_2 = 10 \text{ cm} \). The magnification of the mirror is \( M_2 = -q_2/p_2 = -10/6.67 = -1.5 \). The image is inverted by the mirror.

Finally, this image acts as a real object for the lens, but for light going through the lens from the right to the left. So, the object distance is \( p_3 = 20 \text{ cm} - q_2 = 10 \text{ cm} \). Using the lens equation
\[
\frac{1}{p_3} + \frac{1}{q_3} = \frac{1}{f_{\text{lens}}}
\]
we find that \( q_3 = 10 \text{ cm} \). This is positive, so the image is real and is formed to the left of the lens. The magnification of the lens in this direction is \( M_3 = -q_3/p_3 = -10/10 = -1 \). The image is again inverted.

The overall magnification is \( M = M_1 M_2 M_3 = -2.5 \). The final image is inverted.

### 5. Two Slit Interference

When we derived the 2-slit formula, we assumed that the laser light beam, of wavelength \( \lambda \), was perpendicular to the surface of the screen containing the slits. We found that at angles, \( \theta \), satisfying \( d \sin \theta = m \lambda \) where \( m \) is an integer, and \( d \) is the slit spacing, we would get interference maxima. If we now consider the case where the laser light makes an incident angle \( \phi \) relative to the normal to the screen containing the slit, at what angles does one now get interference maxima? Express your answer in terms of \( d, \phi \) and \( \lambda \).

**SOLUTION:** When the incident light was perpendicular to the screen, the path difference for light rays coming from the 2 slits to a point on the screen was \( \delta = d \sin \theta \). Now there is an additional path difference for the light to get to the slits (since it is no longer normally incident) of \( d \sin \phi \). So we get constructive interference when \( d \sin \theta + d \sin \phi = m \lambda \) or \( \sin \theta = m \lambda /d - \sin \phi \).

### 6. Two Slit Interference

Light composed of two different wavelengths illuminates a double slit, forming two interference patterns that are superimposed on a screen. The fifth-order maximum of one color falls exactly at the location of the third-order maximum of the other color. Calculate the ratio of the two wavelengths.

**SOLUTION:** For one wavelength we have that at the angle \( \theta \), \( d \sin \theta = 5 \lambda_1 \) and for the other wavelength we have that at the same angle \( \theta \), \( d \sin \theta = 3 \lambda_2 \). Equating the two expressions we have that \( \lambda_1 / \lambda_2 = 3/5 \).

### 7. Two Slit Interference

Two narrow slits, 0.25 mm apart are illuminated with light of wavelength \( \lambda \) at normal incidence. The intensity of the light falling on a screen 5 m
away is shown in the following graph, where \( x \) is the distance from the central maximum on the screen.

<table>
<thead>
<tr>
<th>Intensity (W/m²)</th>
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<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
x (cm) & -4 & -2 & 0 & 2 & 4 \\
\hline
\end{array}
\]

SOLUTION: We can use the relation \( d \sin \theta = m \lambda \). The first maximum \( (m = 1) \) occurs at 1cm so \( \sin \theta \approx \theta = 1\text{cm}/5\text{m} = 0.002 \). \( \lambda = d \theta = (2.5 \times 10^{-3}\text{m})(2 \times 10^{-1}) = 500 \text{ nm} \).

a) At \( x = 5 \text{ cm} \), what is the phase difference between the light coming from the two slits?

SOLUTION: The phase difference is \( \delta = 2\pi m \) for the \( m^{th} \) maximum. \( x = 5\text{cm} \) is the fifth maximum and so \( \delta = 10\pi \).

b) What would be the intensity of the light falling on the screen if only one slit is open?

SOLUTION: With both slits open, the average value of the intensity, from the figure, is half the peak, or 4 W/m². With only one slit open, the power is reduced by two, and so the average intensity is also reduced by two. However, with only one slit open, there is no interference pattern, so the average intensity equals the maximum intensity. The maximum intensity is thus 2W/m², a factor of 4 less than with both slits open.

c) Find the wavelength of the light.