We see that at \( t=0 \), when \( p=3 \) m, the image real and is at 0.6 m. As the ball falls and \( p \) decreases, \( q \) increases (the image moves up). The image and the object overlap when

\[
q = \frac{p(1 \text{ m})}{2p - 1 \text{ m}} = p,
\]

which has as its solution \( p=1 \) m. As the ball continues to drop, the image rises and goes to infinity when the ball is 0.5 m above the mirror. At smaller values of \( p \), the image becomes virtual (it appears to be behind the mirror) and moves in from negative infinity, eventually coinciding again with the object when the ball hits the mirror.

(b). From part (a), the ball and its image coincide at \( p=1 \) m. Since the ball is in free fall, \( p(t) = 3 \) m - \((1/2)gt^2\). The time of overlap is given by the solution of \( 1 \text{ m} = 3 \text{ m} - \(1/2)gt^2\), in which \( g = 9.8 \text{ m/s}^2\). The answer is \( t=0.639 \) s.

3. Thin Lens

The image of an object placed 24 cm away from a thin lens forms at a distance of 51 cm on the other side of the lens.

a) What is the focal length?

SOLUTION: We apply the lens equation,

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]

We know that \( p=24 \) cm and \( q=51 \) cm; solving for \( f \) gives \( f=16.32 \) cm.

b) What type of lens is it?

SOLUTION: The focal length is positive, so the lens is a converging lens.
c) Is the image real?

**SOLUTION:**
The image distance is positive, so the image is real.

d) What is the magnification? Is the image upright?

**SOLUTION:** The magnification \( M = -q/p = -51/24 = -2.125 \) is negative, so the image is inverted.

### 4. Optics in a glass rod

A glass rod of refractive index 1.5 is ground and polished at both ends to hemispherical surfaces of 5 cm radius. When an object is placed on the axis of the rod and 20 cm from one end, the final image is formed 40 cm from the opposite end. What is the length \( L \) of the rod?

**SOLUTION:** The first surface will form an image within the glass rod, which will act as the object for the second surface. To find this image, we use the relation for a refracting surface

\[
\frac{n}{p} + \frac{n'}{q} = \frac{n' - n}{R}
\]

In this case, \( n=1 \), \( n'=1.5 \), \( p=20 \text{ cm} \), \( R=5 \text{ cm} \), and we want to find \( q \). Plugging in and solving gives \( q=30 \text{ cm} \). Since \( q \) is positive, the image is formed to the right of the first surface.

This image will act as the object for the second surface. We apply the same equation to find the image for the second surface, which is the final image. In this case, \( n'=1 \), \( n=1.5 \), \( p = L - 30 \text{ cm} \), \( R = 5 \text{ cm} \), and we want to find \( L \). We have

\[
\frac{1.5}{L - 30 \text{ cm}} + \frac{1}{40 \text{ cm}} = -0.5
\]

Solving for \( L \) gives \( L=50 \text{ cm} \).

### 5. Focal point shift

Rays from a lens are converging toward a point image \( P \), as shown in the figure. What thickness, \( t \), of glass of index 1.5 must be interposed, as in the figure, in order that the image shall be formed at \( P'' \)? (Assume that the angles of the rays to \( P \) are small)

**SOLUTION:** Both sides of the glass slab will act as refracting surfaces for the light converging to \( P \); the object-image relation for these surfaces is

\[
\frac{n}{p} + \frac{n'}{q} = 0
\]
since the radius of a flat surface is infinite. We consider first the left surface. The point $P$ acts as a virtual object for this surface, since it is located to the right of the surface. Hence we have for this surface $n=1$, $n'=1.5$, $p = -14.4$ cm, and we want to find $q$. The above equation implies that $q = -p\frac{n'}{n}$. Plugging in gives $q=21.6$ cm. Since $q$ is positive, the image is formed to the right of the surface.

This image then serves as the object for the second surface. Since it’s located to the right of the surface, it’s virtual and $p = \frac{q}{n}$. Plugging in gives $q = 21.6 \text{ cm}$. Since $q$ is positive, the image is formed to the right of the surface.

From geometry we have $\delta = \phi t$, $h = 14.4\theta$ cm, and $h' = (14.7 \text{ cm} - t)\theta$. Since $h = h' + \delta$, we have

\[14.4\theta \text{ cm} = \frac{\theta}{1.5} + (14.7 \text{ cm} - t)\theta\]

which gives $-0.3 \text{ cm} = -\frac{t}{1.5}$ or $t = 0.9$ cm.

6. Two slit interference
A two slit interference experiment is contained in a sealed box which is evacuated. The slits are illuminated with light of wavelength $\lambda$ and an interference pattern is observed on a screen at a distance $L$ from the slits. Then the box is filled with a transparent gas of refractive index $n_{gas}$. Describe the change to the interference pattern.

SOLUTION: The wavelength of the light will decrease when the gas is introduced ($\lambda_{gas} = \lambda_0/n_{gas}$). The separation between the interference fringes is proportional to wavelength, so therefore the separation between the interference fringes will decrease.

7. Double Slit Interference
Laser light of wavelength 600 nm is incident normally upon two very narrow slits 0.30 mm apart. The interference pattern is viewed on a screen 5.0 m from the slits. How far apart are the maxima on the screen?

SOLUTION: The maxima occur when $d\sin\theta = m\lambda$. We have $d = 0.30\text{ mm}$, $\lambda = 600\text{ nm}$ and we can approximate $\sin\theta \approx \theta$ because the screen is very far (5m) away. The angular distance between two adjacent maxima will be $\Delta\theta = \lambda/d = \Delta x/L$ where $\Delta x$ is the separation of adjacent maxima, and $L$ is the distance to the screen. Plugging in the numbers we find $\Delta x = L\Delta\theta = L\lambda/d = (5\text{ m})(6 \times 10^{-7}\text{ m})/(3 \times 10^{-7}\text{ m}) = 1$ cm.