Physics 214 Spring 99—Problem Set 10—Optional Problems

Handout April 6, 1999

1. Photons

A useful relation between the energy \( E \) of a photon and its wavelength \( \lambda \) is \( E\lambda = 1.24 \times 10^3 \) eV-nm. Derive this expression.

\[
E\lambda = h\lambda = h\frac{c}{\lambda} = 1.99 \times 10^{-20} \text{ J}\cdot\text{m}.
\]

We now need to convert to units of eV using \( 1 \) eV which gives \( E\lambda = 1.24 \times 10^3 \) eV-nm.

SOLUTION: We know that
\[
2 \text{ eV} = E_{400} - \phi
\]
\[
1 \text{ eV} = E_{600} - \phi
\]

Subtracting the two equations gives
\[
1 \text{ eV} = E_{400} - E_{600} = h(c/400 \text{ nm} - c/600 \text{ nm}) = \frac{h}{4} \times 10^{15} \text{ s}^{-1}
\]

so we calculate \( h = 4 \times 10^{-15} \) eV-s. To get the work function, we substitute in
\[
\phi = E_{400} - 2 \text{ eV} = \frac{(4 \times 10^{-15})(3 \times 10^8)}{4 \times 10^{-7}} - 2 \text{ eV} = 1 \text{ eV}.
\]

2. Photons

An ultraviolet (UV) light bulb emitting 300 nm light and an infrared (IR) light bulb emitting 800 nm light are each rated at 300 W.

a) Which radiates photons at a greater rate? (Justify your answer.)

SOLUTION: We know that power=energy/time = \( NE_\lambda \), where \( N \) is the rate at which photons are radiated. Both light bulbs have the same power rating, so \( P_{UV} = P_{IR} \) which implies that \( \dot{N}_{UV}E_{UV} = \dot{N}_{IR}E_{IR} \). But for each photon, we know that \( E_{UV} > E_{IR} \) which implies that \( \dot{N}_{IR} > \dot{N}_{UV}: \) the IR photons are radiated at a greater rate.

b) What is the ratio of the number of photons per second produced by the IR to the UV bulb?

SOLUTION: \( \dot{N}_{IR}/\dot{N}_{UV} = E_{UV}/E_{IR} = \lambda_{IR}/\lambda_{UV} = 8/3 = 2.67 \)

3. Photoelectric Effect

In the photoelectric experiment it is observed that with \( \lambda = 400 \) nm, the a stopping potential \( V_0 = 2 \) V is needed, whereas when \( \lambda = 600 \) nm, a stopping potential of \( V_0 = 1 \) V is needed. From this data, calculate the work function for the material and Planck’s constant (note: don’t worry if the Planck’s constant you calculate from this data is different from the accepted value!).

SOLUTION: In order to conserve momentum, the momentum of the H atom after it emits a photon must be equal and opposite to the momentum of the photon. Therefore we have
\[
m_Hv_H = p_e = E_e/c
\]
\[
v_H = E_e/(m_Hc^2)
\]

\( mc^2 \) for the H atom is just \( mc^2 \) of the proton in the nucleus (since compared to that, the electron mass can be ignored) which is about 1 GeV. Therefore we get
\[
v_H = (13.6 \text{ eV}/10^9 \text{ eV})c = 1.35 \times 10^{-8}c = 4.1 \text{ m/s}.
\]

4. Recoil from Photon Emission

You are given a hydrogen atom at rest. It emits a photon of 13.6 eV energy. What is its velocity after the photon emission?

SOLUTION: In order to conserve momentum, the momentum of the H atom after it emits a photon must be equal and opposite to the momentum of the photon. Therefore we have
\[
m_Hv_H = p_e = E_e/c
\]
\[
v_H = E_e/(m_Hc^2)
\]

For the H atom, it is just \( mc^2 \) of the proton in the nucleus (since compared to that, the electron mass can be ignored) which is about 1 GeV. Therefore we get
\[
v_H = (13.6 \text{ eV}/10^9 \text{ eV})c = 1.35 \times 10^{-8}c = 4.1 \text{ m/s}.
\]
5. Compton Effect
A photon of wavelength $\lambda$ Compton scatters off an electron of mass $m$ at rest and comes out at an angle $\theta$. What is the kinetic energy of the scattered electron in terms of $\lambda$, $\theta$, and $m$?

**SOLUTION:** The formula for Compton scattering gives that
$$\lambda' - \lambda = \frac{h}{m c} (1 - \cos \theta)$$

The electron kinetic energy is just the difference between initial and final photon energies:
$$E_e = E_{\gamma f} - E_{\gamma i} = \frac{hc}{\lambda} (1/\lambda - 1/\lambda')$$
After a bit of algebra this can be expressed as
$$E_e = \left(\frac{hc}{\lambda}\right) - \left(\frac{mc^2}{1 - \cos \theta + mc/\hbar}\right)$$

6. Photoelectric Effect
You are doing an experiment on the photoelectric effect. Experimentally you observe that as the photon wavelength is changed from $\lambda_1$ to $\lambda_2$, the stopping potential increases by a factor of 5 from its initial value, $V_0$. What is the work function, $\phi$, of the metal in terms of $\lambda_1$, $\lambda_2$, and fundamental constants?

**SOLUTION:** The two data points give the two conditions:
$$\frac{hc}{\lambda_1} = e V_0 + \phi$$
$$\frac{hc}{\lambda_2} = 5e V_0 + \phi$$
Rearranging the two equations and eliminating $V_0$, we get
$$\phi = \frac{5hc}{4\lambda_1} - \frac{hc}{4\lambda_2}$$

7. Wien displacement law
The wavelength $\lambda_{\text{max}}$ at which the spectral radiance of a blackbody has its maximum value for a particular temperature $T$ is given by the Wien displacement law, $\lambda_{\text{max}} T = 2898 \, \mu m K$. The effective surface temperature of the sun is $5800^\circ K$.

a) At what wavelength would you expect the sun to radiate most strongly if it behaved like a blackbody?
b) Is this in the visible region of the spectrum? If so, what color is it?
c) Does the sun appear to have this color? If not, explain why not.

**SOLUTION:**
a) Using $T = 5800^\circ K$ and the Wien displacement law gives $\lambda_{\text{max}} = 500 \, \mu m$.
b) This is in the visible region of the spectrum; the color is blue-green.
c) The visible range of the human eye is from 400 to 650 nm. The asymmetric thermal radiation distribution from the sun, although peaked at 500 nm, cuts off rapidly on the blue end, and has a long tail on the long-wavelength red end. The color we see is thus shifted from the maximum blue-green toward longer wavelengths; hence the sun appears yellow.