Physics 214 Fall 98—Problem Set 4—[Total Points: 12] Due Before 9:00 am September 24 1998

Homework [4 Points]

1. Reading Assignments from Serway
   Week Beginning September 15: 16.8, 16.6
   Week Beginning September 22: 17.0 - 17.3, 17.5

2. Wave Propagation
   A sawtooth pulse has the shape shown in the figure as it travels at 100 m/s along a stretched wire. At $t = 0$, the leading edge of the pulse is 6 m from a fixed end of the wire. Make a sketch to show the shape of the wire 0.10 s later, with the significant dimensions of the pulse shape indicated in your sketch.

3. Energy and Boundary Conditions
   In the computing part of Problem Set 5 you are going to numerically generate a right travelling pulse, wave speed $v = 2$ m/s and tension $\tau = 8$ N, by considering a string being displaced (measured in cm) at the origin such that
   
   $$y(0, t) = \exp \left[ -\frac{1}{2} (4 - 2t)^2 \right]$$

   a) Write down an analytical expressions for $y(x, t)$.
      [Hint: $y(x, t) = f(t - x/v)$]
   b) Hence determine the the rate at which the string is gaining or losing energy at the origin, i.e., $x = 0$, as a function of time $t$.

4. Traveling Wave on a String - Prelim I Spring 98 (cont.)
   The figure below shows a traveling wave propagating on a string at time $t = 0$. The wave is propagating to the right (positive x-direction). The tension in the string is $\tau = 8$ N, and the string has a mass per unit length of $\mu = 2$ kg/m. The string has a length of 10 m, and has a free end at $x = 10$ m.

   a) Draw a graph of the amplitude of the wave at $t = 4$ sec, as a function of $x$. Label your axes clearly and be sure to give units.
   b) Draw a graph of the transverse velocity (chunk speed) of the wave at $t = 4$ sec, as a function of $x$. Label your axes clearly and be sure to give units.
   c) Draw a graph of the power in the wave at $t = 4$ sec, as a function of $x$. Label your axes clearly and be sure to give units.
5. Reflected and Transmitted Waves - Prelim I Spring 98

A pulse of height \( A_I \), width \( X_I \) travels in a string of mass density \( \mu_1 \) and is incident on a string of mass density \( \mu_2 \). The ratio of mass densities \( \mu_1 : \mu_2 \) is 1:9. The strings are joined together, and have the same tension.

Which picture correctly describes the string after the pulse has interacted with the junction between the two strings? The pictures are drawn to scale. Also explain why the other pictures are incorrect.

\[ \begin{align*}
A_I & \quad \mu_1 \quad \mu_2 \\
X_I &
\end{align*} \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

6. Reflected and Transmitted Waves

Here we will extend the analysis of the reflection and transmission of wave pulses that you are doing in the computer assignment for this week from \textit{pythag}. The goal is to derive an expression for the reflected and transmitted wave amplitudes, and then to test it.

You are given a string with tension \( \tau \). The left half of the string has mass per unit length \( \mu_1 \) and the right half of the string has mass per unit length \( \mu_2 \). Consider a pulse of amplitude \( A_I \) and width \( \Delta x_I \) propagating on the string, incident from the left. When the pulse hits the discontinuity where the mass per unit length changes from \( \mu_1 \) to \( \mu_2 \), part of the pulse is transmitted with amplitude \( A_T \) and width \( \Delta x_T \), and part of the pulse is reflected with amplitude \( A_R \) and width \( \Delta x_R \).

\( \Delta x_T \) and \( \Delta x_R \) in terms of \( \Delta x_I \)

\( a) \) What are \( \Delta x_T \) and \( \Delta x_R \) in terms of \( \Delta x_I \)?

\( b) \) Use the fact that the string is continuous at the interface to derive a relation between \( A_I \), \( A_R \), and \( A_T \).

c) Energy conservation leads to the conclusion that the energy must leave the discontinuity at the same rate at which it enters it. That is, \( P_I + P_R = P_T \). Use the approximate expressions for the power (derived in lecture by dimensional analysis) to derive the following expressions for \( A_R \) and \( A_T \) in terms of \( A_I \) and the wave speeds in the two halves of the string:

\[ A_T = \frac{2v_2}{v_1 + v_2} A_I \]

\[ A_R = \frac{v_2 - v_1}{v_2 + v_1} A_I \]

d) Using the incident amplitude \( A_I \) and mass densities \( \mu_1 \) and \( \mu_2 \) you recorded in \textit{pythag}, parts P4(a) and P4(b), calculate what the transmitted and reflected amplitudes \( A_T \) and \( A_R \) should be for the step down and step up in density. Compare these with the transmitted and reflected amplitudes you recorded in the lab. Do they agree?

\textbf{Computing} \hspace{1cm} [8 Points]

1. \textit{Pythag}

The following sections from the lab manual for \textit{pythag} needs to be completed and handed in:

\begin{itemize}
  \item Section P3 : Boundary Conditions and Colliding Pulses
  \item Section P4 : Reflection and Transmission
\end{itemize}

Before starting please read the Introduction and Introduction to the Simulation sections.