Physics 214 Fall 98—Problem Set 11—[Total Points: 10] Due Before 9:00 am November 12 1998

Handout November 3 1998

Homework [5 Points]

1. Reading Assignments from Serway
   Week Beginning November 3: 41.6 - 41.7
   Week Beginning November 10: 41.8 - 41.10; 42.0 - 42.6

2. Time-dependent Schrödinger Equation
   a) Show that \( \Psi(x,t) = Ae^{i(kx-\omega t)} \) satisfies the wave equation for a string:
      \[
      \frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi(x,t)}{\partial t^2}
      \]
   b) Show that the same wave function \( \Psi(x,t) = Ae^{i(kx-\omega t)} \) satisfies the time-dependent Schrödinger equation with \( V(x) = 0 \):
      \[
      -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}
      \]
   c) Show that \( \Psi(x,t) = A\cos(kx-\omega t) \) does not satisfy the time-dependent Schrödinger equation with \( V(x) = 0 \).

3. Particle in a Box
   Consider an electron of mass \( m \) trapped in the following potential:
   
   Sketch the wavefunctions for the three lowest standing wave states with the energies \( E_1, E_2, E_3 \).
   b) For the first excited state, of energy \( E_2 \), where is it most probable to find the electron?
   c) How will the energy of the lowest energy state change if the width of the box is reduced by a factor of 2?
   d) What is the probability that you will find the electron between \(-L/2\) and \(+L/2\) when the electron is in energy state \( E_n \) and \( n \) is even?
   e) If the electron is in its ground state, (with energy \( E_1 \), what is the probability that it will be found in the interval between \( x = 0 \) and \( x = \Delta x \), where \( \Delta x \ll L \)?

4. Finite Step Potential
   You are given a 1-dimensional potential with \( U(x) = 0 \) for \( x \leq 0 \) and \( U(x) = U_0 \) for \( x > 0 \) where \( U_0 \) is a positive constant. See Figure
   
   Sketch what a non-trivial solution \( \psi(x) \) vs \( x \) to Schrödinger's equation would look like for a particle with energy \( E \) where:
   a) \( E < U_0 \).
   b) \( E = 2U_0 \).
   c) \( E > U_0 \).
5. Particle in a Potential Well (Qualitative)

Consider an electron of mass \( m \) trapped in the following potential well:

[a] Sketch the wave functions \( \psi_1 \) and \( \psi_2 \) for the two lowest bound standing wave states with the energies \( E_1 \) and \( E_2 \).

[b] Does the lowest energy ("ground") state, \( E_1 \), have a lower or higher energy than the lowest energy state of an electron in a box of the same width \( L \)? Why?

c) Sketch the wave function if the electron has an energy of \( 4V_0/3 \).

6. Schrödinger Equation - Prelim III Spring 98

A particle of mass \( m \) moves in a potential well of width 2 (from \( x = -1 \) to \( x = 1 \)), and in this well the potential is given by

\[
U(x) = \begin{cases} 
\infty & x < -1 \\
-\frac{R^2}{m} \left( \frac{x^2}{1-x^2} \right) & -1 \leq x \leq 1 \\
\infty & x > 1 
\end{cases}
\]

In addition, the particle is in a stationary state described by the wave function

\[
\psi(x) = \begin{cases} 
0 & x \leq -1 \\
A (1 - x^2) & -1 < x < 1 \\
0 & x \geq 1 
\end{cases}
\]

[a] Determine the energy of the particle in terms of \( h \) and \( m \).

[b] Determine the numerical value of \( A \).

[c] Determine the most probable location(s) of the particle.

[5 Points]

1. Schrödgr

The following section from the lab manual for \textit{schrödgr} needs to be completed and handed in:

- Section S1: Infinite Square Well

Before starting please read the Introduction and Introduction to the Simulation sections.