**Quiz #8: Interference and Antenna Arrays**

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1. **Array of Antennas.** 16 antennas are arranged in an East-West row, in a straight line between Elmira (West) and Binghamton (East).

   The radio station transmits at \( f = 100 \text{ kHz} \). The antennas are spaced 0.75 km apart. Each antenna transmits a phase \( \phi_0 \) of the antenna to its immediate West: that is, if the one closest to Elmira is transmitting \( \cos(2\pi ft) \), the next one is transmitting \( \cos(2\pi ft + \phi_0) \), and so on.

   **(A)** What phase difference \( \phi_0 \) should the station use if they want to focus their transmission toward Elmira?

   Let Elmira be a distance \( x \) from the nearest antenna.

   \[
   \phi_0 = \frac{\pi}{2}
   \]

   The wave from the nearest antenna as it hits Elmira is the same phase as the wave from that antenna at time \( t - \frac{x}{c} \): \( \cos(2\pi f (t - \frac{x}{c})) \).

   The wave from, e.g., the next antenna is likewise \( \cos(2\pi f (t - \frac{x+750}{c}) + \phi_0) \).

   For constructive interference,

   \[
   2\pi f (t - \frac{x}{c}) = 2\pi f (t - \frac{x+750}{c}) + \phi_0
   \]

   \[
   \phi_0 = 2\pi f \left( \frac{750}{c} \right) = 2\pi \left( \frac{10^5}{3 \times 10^8} \right) \left( \frac{750}{c} \right)
   \]

   \[
   = \frac{\pi}{2}
   \]
(B) How big an angle will the main transmission beam subtend? (That is, what is twice the angle $\alpha_{\text{min}}$ to the first minimum in the intensity pattern?)

\[ 750\cos(\alpha_{\text{min}}) \]

[Note: $\alpha_{\text{min}}$ is not measured from perpendicular, \( \cos, \text{not} \sin \)]

At a distance $x$ from the nearest antenna, the rest antenna gives $2\pi f (t - \frac{x}{c})$ and the next gives $2\pi f \left( \frac{t - x}{c} - \frac{750\cos x}{c} \right) + \phi_0$.

The phase difference is

\[ -\frac{\pi}{2} \cos \alpha_{\text{min}} + \phi_0 \]

\[ = -\frac{\pi}{2} \cos \alpha_{\text{min}} + \frac{\pi}{2} \]

\[ 2\alpha_{\text{min}} = 82.8^\circ \]

\[ -\frac{\pi}{2} \cos \alpha_{\text{min}} + \frac{\pi}{2} = \frac{2\pi}{16} \]

\[ \cos \alpha_{\text{min}} = 1 + \left( \frac{2\pi}{16} \right) \left( \frac{2}{\pi} \right) = 1 + \frac{1}{4} = \frac{5}{4} \left( \text{not possible} \right) \]

\[ \alpha_{\text{min}} = \arccos \left( \frac{3}{4} \right) = 0.72 \text{ radians} = 41.4^\circ \]

For destructive interference, this should equal $2\pi/16$ (for 16 antennas). $\alpha_{\text{min}} = \frac{3\pi}{4}$. 

\[ \text{For destructive interference, this should equal} \]

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