Quiz #6: Standing Waves and Resonance
Jim Sethna

1. Standing Waves and Nodes. You are given a rope of tension $\tau$ and mass per unit length $\mu$. You measure a resonance with $n$ nodes at a frequency $f_n$ cycles/second. Your lab partner then slides the adjustment too fast, and you miss the next resonance: the two of you then measure a resonance with $n+2$ modes at a frequency $f_{n+2}$ cycles per second. In your notes, you neglect to write down the value of $n$, the number of nodes. Solve for the length of the rope $L$, in terms of $\tau$, $\mu$, $f_n$, and $f_{n+2}$, (but not involving $n$)!

$$\lambda = \frac{2L}{n} \quad \lambda f = \sqrt{\frac{\tau}{\mu}}$$

$$f_n = \sqrt{\frac{\tau}{\mu}} \frac{n}{2L} \quad f_{n+2} = \sqrt{\frac{\tau}{\mu}} \frac{n+2}{2L}$$

$$f_{n+2} - f_n = \sqrt{\frac{\tau}{\mu}} \frac{1}{L} \quad L = \frac{\tau}{\mu} \left( f_{n+2} - f_n \right)$$

$$L = \frac{1}{\sqrt{\frac{\tau}{\mu} \left( f_{n+2} - f_n \right)}}$$

2. Standing Wave on a Viola String. A viola player is playing A above middle C (at 440 Hz). She wants to drop down in pitch to middle C (a bit below 262 Hz). The vibrating piece of the viola string, stretching from the bridge to her finger, is 25 cm while she is playing the note A. If she wants to play the note C on the same string, how far should she move her finger? In what direction (toward the bridge shortening the string, or away from the bridge lengthening the string)?

$$L = 25$$

$$L = 25 \left( \frac{440}{262} \right) = 44$$

$$L = 26 \left( \frac{440}{262} \right) = 44$$

$$L = \frac{26 \left( \frac{440}{262} \right)}{26} = 12 \quad L' - L = 44 - 26 = 18 \text{ cm}$$

Distance = $25 \left( \frac{178}{262} \right)$, Direction (circle one): Towards Away
3. **Energy Loss at Resonance.** (HARD: from Optional 6.14)

Any real system has mechanisms for energy loss. Imagine a system of a string (fixed at both ends) that you are going to be injecting pulses onto and that you are trying to drive in a resonant fashion with frequency \( f_1 = v/2L \). Let’s assume that the system has some loss factor \( \alpha \) so that when you inject a pulse and it goes down the string and then comes back, at the point when you are ready to inject a second pulse, the amplitude of the first pulse has decreased from an initial value of \( A \) to \( \alpha A \). Every subsequent round trip of a pulse on the string decreases its amplitude by an additional factor \( \alpha \). What is the maximum amplitude that the pulse will build up to, after an infinite number of injected pulses of amplitude \( A \)?

\[
\begin{align*}
A & \\
A + \alpha A & \\
A + \alpha A + \alpha^2 A & \\
A \left(1 + \alpha + \alpha^2 + \ldots\right) &= \frac{A}{1 - \alpha}
\end{align*}
\]