Multiple Choice

1. (5 pts)

The minute gradations on a watch are 2 mm apart. With perfect eyesight, how far away from the watch can you be in 550 nm wavelength light to just resolve them? Take the diameter of your pupil to be 0.5 cm.

\[ \sin \theta = \frac{2}{a} = 1.22 \frac{0.55 \times 10^{-6} \text{ m}}{0.5 \times 10^{-2} \text{ m}} \approx 1.3 \times 10^{-4} \]

- (A) 2.75 m
- (B) 15.0 m
- (C) 100 m
- (D) 550 m
- (E) 1100 m

2. (5 pts)

The one-dimensional potential \( V(x) \) shown is known to have only one bound state for an electron, mass \( m \). A particular electron in this potential has energy \( E \) slightly less than \( V_0 \).

![Diagram of potential function with nodes and discontinuities]

Which of the following sketches gives the best representation of the electron's wave function \( \psi(x) \)?

- (A) Node implies a state of lower energy exists
- (B) Wrong curvature when \( x > a \)
- (C) \( V \) finite, discontinuous slope
- (D) Wrong curvature when \( x > a \)
3. (5 pts)

How many photons per second are emitted by a laser that delivers a beam of photons of wavelength $\lambda$, frequency $f$, if the total power of the laser beam is $P$?

(A) $P\lambda/(hc)$
(B) $f$
(C) $hc/(P\lambda)$
(D) $hf/P$
(E) $mc^2/hf$
(F) none

\[
\frac{\text{number of photons}}{\sec} = \frac{\text{energy}}{\frac{1}{\sec}} \cdot \frac{1}{\text{photons/energy}} = P\frac{1}{hf} = P\frac{\lambda}{hc}
\]

4. (5 pts)

A photon of energy $E$ Compton scatters straight back ($\theta = \pi$) from an electron, rest mass $m_0$, initially at rest. What is the final energy $E'$ of the scattered photon?

(A) $m_0c^2/2$
(B) $m_0c^2/(2E + m_0c^2)$
(C) $h/(2m_0c)$
(D) $(2/(m_0c^2) + 1/E)^{-1}$
(E) $m_0c^2$
(F) $2h/m_0c$

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta) = \frac{2h}{mc}
\]

\[
\frac{\hbar c}{E'} - \frac{\hbar c}{E} = \frac{2\hbar}{mc}
\]

\[
\frac{1}{E'} = \frac{2}{mc^2} + \frac{1}{E}
\]

\[
E' = \left(\frac{2}{mc^2} + \frac{1}{E}\right)^{-1}
\]
Short Answer

1. (15 pts)

Using light of wavelength \( \lambda \), an interference pattern is observed from three identical narrow slits. The spacings between slits is \( d \) and the slit widths are narrow enough to be neglected. With only one slit open the light intensity of the central maximum is \( I_0 \). Let \( I(\theta) \) be the intensity at angle \( \theta \) with all three slits open. Though the central intensity exceeds \( I_0 \) with all slits open, it is observed that there are some angles at which the intensity is \( I_0 \).

a) \( 5 \text{ pts} \) In terms of \( I_0 \), determine \( I(0) \), the intensity at \( \theta = 0 \) with all slits open.

\[
\frac{I(0)}{I_0} = \left( \frac{E(0)}{E_0} \right)^2 = 3^2 = 9
\]

\[
I(0) = 9I_0
\]

b) \( 10 \text{ pts} \) Draw two phasor diagrams representing the light amplitudes at angles \( \theta_1 \) and \( \theta_2 \), the smallest angles (both positive) for which \( I(\theta_1) = I(\theta_2) = I_0 \).

One slit: \[
\begin{align*}
\text{Sum:} & \quad E_0 \\
\end{align*}
\]

Three slits: \[
\begin{align*}
\text{Sum:} & \quad E_0 \\
\end{align*}
\]
2. (20 pts)

Three slits, with widths $d$, $3d$, and $d$ respectively, are so closely spaced that in effect they become a single slit of width $5d$. With light of wavelength $\lambda$ the first single slit diffraction minimum is observed at angle $\theta_1$. For the sketches asked for in the following parts you may assume that all angles are small, that horizontal scales are indicated in terms of $\theta_1$, and that vertical scales are arbitrary.

a) (5 pts) Find the angle $\theta_1$.

$$\theta_1 = \frac{\lambda}{5d}$$

b) (5 pts) Sketch the observed single slit intensity pattern $I(\theta)$ for the range from $\theta = 0$ to $\theta = 2\theta_1$. If there are minima, be sure they are sketched at approximately the correct positions along the axis, and labeled.

\[ \text{Sketch of single slit intensity pattern.} \]

\[ \theta_1 \]

\[ 2\theta_1 \]

\[ \text{Minima at} \theta = \frac{\lambda}{3d} \text{and} \frac{5\lambda}{3d} \]

c) (5 pts) Over the same range from $\theta = 0$ to $\theta = 2\theta_1$ as in part (b) sketch the intensity pattern that would be observed with the two outer slits blocked off (leaving a single slit of width $3d$.) If there are minima, label their positions.

\[ \text{Sketch of single slit intensity pattern.} \]

\[ \theta_1 \]

\[ 2\theta_1 \]

\[ \frac{\lambda}{3d} \]

\[ \frac{5\lambda}{3d} \]

\[ \theta_1 \]

d) (5 pts) With the central slit covered and the outer two slits uncovered, one has two slits with separation $4d$. Using the approximation that $d << 4d$, sketch the resulting double slit pattern. Use the same range as in the two previous parts, from $\theta = 0$ to $\theta = 2\theta_1$. If there are minima, label their positions.

\[ \text{Sketch of double slit intensity pattern.} \]

\[ \theta_1 \]

\[ 2\theta_1 \]

\[ \frac{\lambda}{4d} \]

\[ \frac{5\lambda}{4d} \]

\[ \theta_1 \]

\[ \theta_2 \]

\[ \theta_3 \]

\[ \theta_1 \]

\[ \theta_2 \]

\[ \theta_3 \]
3. (10 pts)
A person hears nine loudspeakers, each emitting sound of exactly the same, unvarying, intensity, and approximately the same frequency. The actual frequencies are 399.2, 399.4, 399.6, 399.8, 400.0, 400.2, 400.4, 400.6, and 400.8 Hz. Initially, at \( t = 0 \) the sound waves are all in phase and the person hears a very loud sound. The waves gradually get out of phase, and for the first time, at \( t = T_1 \), the intensity is zero. At time \( t = T_2 \), for the first time, the sound again becomes as loud as at \( t = 0 \). Determine \( T_1 \) and \( T_2 \).

\[
\Delta \omega T_1 = \frac{2\pi}{9} \quad \Rightarrow \quad T_1 = \frac{5}{9} \text{ sec}
\]

\[
\Delta \omega T_2 = \frac{2\pi}{9} \quad \Rightarrow \quad T_2 = 5\text{ sec}
\]

4. (10 pts)
A particle of rest energy 4 MeV has momentum 3 MeV/c. Give the total energy \( E \) of the particle as well as its speed \( v \), as a ratio \( v/c \), and its relativistic factor \( \gamma \).

\[
E = \sqrt{\rho^2 c^2 + m_0^2 c^4}
\]
\[
= \sqrt{3^2 + 4^2} = 5\text{ MeV}
\]

\[
\frac{v}{c} = \frac{\rho c}{E} = \frac{3}{5}
\]
\[
v = 0.6
\]
\[
\gamma = \frac{E}{m_0 c^2} = \frac{5}{4} = 1.25
\]
\[
\gamma = 1.25
\]
\[
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.25
\]
\[
\frac{1}{\sqrt{1 - 0.6^2}} = 1.25
\]
5. (10 pts)

A particle of mass \( m \) is bound in a 1-dimensional infinite potential square well of width \( L \). (The available energy levels for this system are given in the list of formulas at the back of the exam, for \( n = 1, 2, \ldots \)) An external influence occasionally excites the particle to one of the higher states and then it decays back down by the emission of photons. Find \( \lambda \), the longest wavelength observed for emitted photons.

\[
E_n = \frac{\hbar^2 n^2}{8 m L^2},
\]

\[
\Delta E_{\text{min}} = \frac{\hbar^2}{8 m L^2} (4 - 1)
\]

\[
\lambda_{\text{max}} = \frac{\lambda}{\Delta E_{\text{min}}} = \frac{\hbar c}{8 m L^2} \frac{1}{\sqrt{3}}
\]

\[
\text{smallest energy diff.} \quad \lambda = \frac{8 m L^2 c}{3 \hbar}
\]

6. (15 pts)

A particle of mass \( m \) is confined to the \( x \)-axis by a 1-dimensional potential \( V(x) \). The particle is in a stationary state of energy \( E \) and its wave function is

\[
\psi(x) = Ae^{-\frac{x^2}{2\sigma^2}}
\]

where \( A \) and \( \sigma \) are constants.

\[
\text{in terms of}
\]

a) (4 pts) Express the constant \( A \) as an integral. (Don't attempt to evaluate the integral.)

\[
A = \left( \int_{-\infty}^{\infty} -\frac{x^2}{2\sigma^2} \right)^{-\frac{1}{2}}
\]

b) (4 pts) \( P(|x| > \sigma) \) is the probability that the particle is more distant than \( \sigma \) from the origin. Express \( P(|x| > \sigma) \) as an integral. (Don't attempt to evaluate the integral.)

\[
\psi(x) = \psi(-x) \quad P(x) = P(-x)
\]

\[
P(|x| > \sigma) = \frac{2A}{\sigma} \int_{\sigma}^{\infty} -\frac{x^2}{2\sigma^2} dx
\]

\[
c) (7 pts) \text{Using the wave function given above and the time-independent Schrödinger equation, find } E - V(x).
\]

\[
\frac{d\psi}{dx} = -\frac{\hbar^2}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \frac{d^2\psi}{dx^2} = -\frac{A}{\sigma} e^{-\frac{x^2}{2\sigma^2}} + \frac{A}{\sigma^2} e^{-\frac{2x^2}{2\sigma^2}}
\]

\[
E - V(x) = 2m \frac{\hbar^2}{\sigma} \frac{d^2\psi}{dx^2} = -\frac{1}{\sigma^2}(1 - \frac{x^2}{\sigma^2}) \psi_x
\]

\[
E - V(x) = \frac{\hbar^2}{2m} \frac{1}{\sigma^2}(1 - \frac{x^2}{\sigma^2})
\]