Counting photons inside a box

Consider a box of volume $V$ containing vacuum with its electromagnetic field in equilibrium at temperature $T$.

(a). In the limit of small frequency, is the number of photons per mode divergent? What about the energy per mode?

(b). Estimate (within a constant of order unity) the total number of photons per unit volume, $N/V$, using the formulas from the text. This is a “back of envelope” job, not a “heavy algebra” job, so you can brutally approximate the Bose Einstein distribution in the crudest way: divide the range of $\omega$ into two intervals: “small” $(0, \omega_*)$ and “large” $(\omega_*, \infty)$. Then, use the functional forms valid in the “very small” and “very large” limits.

Hint 1: in the limit $\hbar \omega/k_B T \to \infty$, the Bose-Einstein distribution goes to zero.

Hint 2: two handy combinations of variables are the thermal wavelength of light, $\lambda_T \equiv hc/k_B T$, and $x \equiv \hbar \omega/k_B T$.

(c). At room temperature, how many photons are there in a cube 1 mm$^3$, within an order of magnitude?

Hint: room temperature is $1/40$ eV, and $hc \approx 1240$ eV-nm.
(d). OPTIONAL. Consider instead the EM field inside a coax cable or stripline, which is described the same way except that the spatial dependence is one dimensional, and hence the wavevector has only one component. Without working out any details, do you think number of modes per unit length \( N/L \) behaves analogously to \( N/V \) in part (b), or is it qualitatively different?