Mean Field Theory
Jim Sethna, Physics 7653, Fall 2018

Ignores fluctuations (or assumes they are short-range)
• Fluctuations on all scales near (many) critical points
Captures behavior in
• High spatial dimension $> d_{UCD}$
  • Tricritical Ising $d_{UCD} = 3$
  • Jamming $d_{UCD} = 2$
  • Percolation, Random-field Ising model $d_{UCD} = 6$
• Infinite-range systems
• Long-range interactions
• Rough models for inhomogeneous systems, defects
High dimensions

Mean-field theory above the Upper Critical Dimension

$$\mathcal{H} = -J \sum S_i \cdot S_j - H \cdot \sum S_i$$

High $T$: melted
Low $T$: ordered
Critical: $T_c, H=0$

$d_{UCD} = 4$
4-$\varepsilon$ expansion
logs, PowerLogs

$d_{LCD} = 2$ (or 1)
exponentials, jumps, …

Bethe lattice

Mean Field Theory

SARW (self-avoiding random walk)

Ising

XY

Heisenberg

No transition

Spherical

Spin components $n$
Infinite-range interactions
Every site interacts with all/many others

Superconductors:
  Cooper pairs overlap thousands of neighbors
Sync (Strogatz):
  Every firefly sees all others
Laser onset criticality (Exercise 12.6):
  Photons in nonlocal cavity modes
Neurons in brain
  Axons long, 1000’s of connections
Computer science (Selman and Gomes)
  3SAT (Exercise 1.8), NP complete
Physics ‘cavity’ methods winners
Long-range interactions

Power-law interactions between sites make $d_{UCD}=3$

Dipolar interactions between magnetic spins
- Not important near $T_c$ for thermal transitions ($M$ vanishes, screening)
- Important for noise in hysteresis (Barkhausen noise, avalanches)

Elastic interactions between slips
- Earthquake faults
- Dislocations in crystals (crackling noise)
- Shear transformation zones in amorphous metals
Rough guide to behavior
Mean-field theory as a practical, but uncontrolled, approximation

Curie-Weiss for ferromagnets (Cardy)
Van der Waals for liquids and gases
Sherrington-Kirkpatrick for spin glasses
• Ultrametricity
• Replica symmetry breaking
• Cavity method
Avalanche size distribution (Exercise 12.25)
Ginzburg-Landau theories (Exercise 9.5)
• Starting point for ε-expansions