Scaling beyond scaling:
Mean-Field temporal average avalanche shapes away from criticality

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Critical behavior of avalanches in magnets

Magnetic avalanches in real space

Typical avalanche velocity as a function of time

\[ \langle V(t, T) \rangle = T^{1/\sigma v z - 1} V(t/T) \]
Novel mean-field descriptions of avalanches

- **Basic viewpoint:** Shells of “active” degrees of freedom. Shells flip with distinct dynamics
- **Stochastic equation** for shell flipping mean-field description (infinite range)
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  \frac{dV_t}{dt} = c - kV_t + \sqrt{2V} \xi(t)
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\[
\langle V(t, T) \rangle = \lim_{\epsilon \to 0} \frac{\int_0^\infty dV V P(V, t; \epsilon, 0) P(\epsilon, t; V, T)}{\int_0^\infty dV P(V, t; \epsilon, 0) P(V, t; \epsilon, T)}
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\[c = 0, \ k = 0\]
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\langle V(t, \lambda) \rangle = \frac{1}{8}T\lambda(1 - \lambda)
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\[
\lambda \equiv t/T \in [0, 1]
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$$

- **Equations**:
  
  1. \( c = 0, k = 0 \)
  
  2. \( \langle V(t, \lambda) \rangle = \frac{1}{8} T\lambda (1 - \lambda) \)

  \( \lambda \equiv t/T \in [0, 1] \)
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- Stochastic equation for shell flipping mean-field description (infinite range)

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\[c = 0\]

\[
\langle V(T, \lambda) \rangle_k = \frac{1}{2k} \frac{(e^{kT(1-\lambda)} - 1)(e^{kT\lambda} - 1)}{e^{kT} - 1}
\]

\[
= \langle V(T, \lambda) \rangle_p - \frac{1}{24} k^2 T^3 (1 - \lambda)^2 \lambda^2 + \mathcal{O}(k^4).
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Symmetric shapes from experiments on thin films

- thin films → no eddy currents’ distortions
- Symmetric shapes but spurious oscillations
Comparing theory with experiments

• Demagnetizing effects in **mean-field** theory and experiments away from criticality

• still a small **asymmetry**, but from where?
Comparing theory with experiments

- Agreement for distributions of sizes and durations
Background noise and its horrendous effects

...from the background

average power spectrum plagued with spurious peaks

...to the apparatus response!
Sneaky way to remove the effects of the external noise

- Apply Wiener filtering on the timeseries with three ingredients: impulse function, external noise, estimated signal,

\[
\tilde{S}(f) = \frac{R(f)}{h(f)} \frac{|S(f)|^2}{|N(f)|^2 + |S(f)|^2}
\]
How filtering affects scaling predictions?

- Investigate effects in simulations of the mean-field model.

  - Differences noticeable when intensity, duration or size are small...
Small asymmetry and the impulse function

- Calculate shapes with/without **background noise and filters**

- **Asymmetry** appears for the **noisy** shapes...

- It is typically removed after filtering
Conclusions

• **Novel techniques** for analyzing experimental timeseries of avalanche behavior,

• **Experimental verification of theoretical predictions** beyond the scaling regime