The actual simulations were carried out in a square shaped cell of linear size $L$ with periodic boundary conditions. The particles were represented by points moving continuously (off lattice) on the plane. We used the interaction radius $r$ as the unit to measure distances $(r = 1)$, while the time unit $\Delta t = 1$ was the time interval between two updatings of the directions and positions. In most of our simulations we used the simplest initial conditions: (i) at time $t = 0$, $N$ particles were randomly distributed in the cell and (ii) had the same absolute velocity $v$ and (iii) randomly distributed directions $\theta$. the velocities $\{v_i\}$ of the particles were determined simultaneously at each time step, and the position of the $i$th particle updated according to

$$x_i(t + 1) = x_i(t) + v_i(t) \Delta t .$$  

(1)

Here the velocity of a particle $v_i(t + 1)$ was constructed to have an absolute value $v$ and a direction given by the angle $\theta(t + 1)$. This angle was obtained from the expression

$$\theta(t + 1) = \langle \theta(t) \rangle_r + \Delta \theta ,$$

(2)

where $\langle \theta(t) \rangle_r$ denotes the average direction of the velocities of particles (including particle $i$) being within a circle of radius $r$ surrounding the given particle. The averaged direction was given by the angle $\arctan[\sin(\langle \theta(t) \rangle_r)/\cos(\langle \theta(t) \rangle_r)]$. In Eq. (2) $\Delta \theta$ is a random number chosen with a uniform probability from the interval $[-\eta/2, \eta/2]$. Thus the term $\Delta \theta$ represents noise, which we shall use as a temperature like variable. Correspondingly, there are three free parameters for a given system size: $\eta$, $\rho$, and $v$, where $v$ is the distance a particle makes between two updatings.

We have chosen this realization because of its simplicity, however, there may be several more interesting alternatives of implementing the main rules of the model. In particular, the absolute value of the velocities does not have to be fixed, one can introduce further kinds of particle interactions and or consider lattice alternatives of the model. In the rest of this paper we shall concentrate on the simplest version, described above, and investigate the nontrivial behavior of the transport properties as the two basic parameters of the model, the noise $\eta$ and the density $\rho = N/L^2$, are varied. We used $v = 0.03$ in the simulations we report on for the following reasons. In the limit $v \to 0$ the particles do not move and the model becomes an analog of the well-known XY model. For $v \to \infty$ the particles become completely mixed between two updates, and this limit corresponds to the so-called mean-field behavior of a ferromagnet. We use $v = 0.03$ for which the particles always interact with their actual neighbors and move fast enough to change the configuration after a few updates of the directions. According to our simulations, in a wide range of the velocities ($0.003 < v < 0.3$), the actual value of $v$ does not affect the results.

![Fig. 1](image)

In this figure the velocities of the particles are displayed for varying values of the density and the noise. The actual velocity of a particle is indicated by a small arrow, while their trajectory for the last 20 time steps is shown by a short continuous curve. The number of particles is $N = 300$ in each case. (a) $t = 0$, $L = 7$, $\eta = 2.0$. (b) For small densities and noise the particles tend to form groups moving coherently in random directions, here $L = 25$, $\eta = 0.1$. (c) After some time at higher densities and noise ($L = 7$, $\eta = 2.0$) the particles move randomly with some correlation. (d) For higher density and small noise ($L = 5$, $\eta = 0.1$) the motion becomes ordered. All of our results shown in Figs. 1–3 were obtained from simulations in which $v$ was set to be equal to 0.03.

Figures 1(a)–1(d) demonstrate the velocity fields during runs with various selections for the value of the parameters $\rho$ and $\eta$. The actual velocity of a particle is indicated by a small arrow, while their trajectory for the last 20 time steps is shown by a short continuous curve. (a) At $t = 0$ the positions and the direction of velocities are distributed randomly. (b) For small densities and noise the particles tend to form groups moving coherently in random directions. (c) At higher densities and noise the particles move randomly with some correlation. (d) Perhaps the most interesting case is when the density is large and the noise is small; in this case the motion becomes ordered on a macroscopic scale and all of the particles tend to move in the same spontaneously selected direction.

This kinetic phase transition is due to the fact that the particles are driven with a constant absolute velocity; thus, unlike standard physical systems in our case, the net momentum of the interacting particles is not conserved during collision. We have studied in detail the nature of this transition by determining the absolute value of the average normalized velocity

$$v_0 = \frac{1}{Nv} \sum_{i=1}^{N} v_i$$

(3)