Superconductivity

BCS Theory: Fermi Sea is unstable to attractive \( \sigma^- \sigma^- \) interactions

Forms Cooper pairs

\[
\varepsilon_{\text{pair}} = \varepsilon_F - \hbar \omega_c \exp\left\{ -\frac{1}{(\varepsilon_0 + \varepsilon)^2} \right\}
\]

\( \varepsilon_0 \) = Fermi Energy
\( \omega_c \) = Debye Frequency
\( \varepsilon \) = Interatomic Strength

\( \varepsilon_0 < 0 \) (attraction)

At low temperature, all free electrons condense into Cooper pairs

\[
T_c \approx \hbar \omega_c \exp\left\{ -\frac{1}{(\varepsilon_0 + \varepsilon)^2} \right\}
\]

(Typically very low temp)

\[
= 0 + 0/\varepsilon_0 + O(\varepsilon_0^2) + \ldots
\]

This was not understood for many years.

Perturbation theory predicts \( T_c = 0 \) to ALL orders.

Given as a "proof" that S.C. cannot occur (TLC).

(Actually \( T_c \) has an essential singularity at \( |\varepsilon| = \varepsilon_0 \))
Morals of the story:

1. Proving something to all orders in pert. theory does not make it true.

2. This is obvious, since we are at a phase transition (i.e. pert. theory only works within phases).

This was not well-known at the time this was being studied.

R.G. Picture:

Interpretation: \( \Lambda \) = Band Width, think Tight Binding Model

Results:

Fixed Point: Non-interacting Fermi Gas

\[ V > 0 \] (Repulsive Interactions) shrink much, cause grainning

\[ V < 0 \] (Attractive Interactions) grow under, cause grainning

\[ \Rightarrow \] a new phase
R.G. Equations:

\[
\frac{dT}{dt} = aT \quad (a > 0)
\]  \hspace{1cm} (1)

\[
\frac{dV}{dt} = ? \quad V \quad (How \ do \ we \ get \ observed \ behavior?)
\]

\[
= -bV^2 \quad (b > 0)
\]  \hspace{1cm} (2)

\(V\) is marginal: Does not grow/shrink to linear order.

Solve Equations:

\[
\frac{dT}{T} = a \, dt \Rightarrow \log \frac{T}{T_0} = at
\]

\[
- \frac{dV}{V^2} = b \, dt \Rightarrow \frac{1}{V} = bt
\]

\[
\Rightarrow T = T_0 \exp \left\{ \frac{a}{b} V^2 \right\} = T_0 \exp \left\{ -\frac{a}{b} V^2 \right\}
\]

High T Fermi Gas (attractive interactions).
So deriving (1) + (2) will give us this qualitative behavior.

Other details:

Fermi F-function \( \frac{1}{F} \) to this picture.

\[
\frac{dF}{dt} = 0 \quad \forall t
\]

To all orders!

Redundant = Marginal to all orders.

(Other examples include:

Ising anisotropy

Central limit Theorem: Gaussian Width)