This talk is divided into two parts. In the first part, I will briefly discuss how disorder can be incorporated into the pure Ising model. We call these systems random magnets.


In the second part of the talk, I will present a heuristic argument, due to A.B. Harris, which addresses the question "Under what conditions will the critical behavior of a disordered system be the same as that of the corresponding pure system. This condition is called the Harris criterion."
References:


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PART I: RANDOM MAGNETS

1. The study of magnetic systems has given us insights into
   - Collective phenomena
   - Universality
   - Broken symmetry
   - Scaling near continuous phase transitions
   - Renormalization group

2. Reasons
   - A large number of experimentally accessible magnetic systems
   - Very simple models of magnetism (like the Ising model) often capture the essential physics of the phases and the ordering transitions of much more complicated systems.

3. Initially the study of collective behavior of magnets focused on ideal pure systems or perfect crystals. Experimentally, one strove hard to eliminate disorder from...
their samples to compare their results with the theory of ideal crystals.

4. However, quenched or frozen-in disorder is ubiquitous and affects the properties of virtually all experimental systems to some degree.

In the 70's and the 80's, people began to realize that disorder might play a fundamental role in the physics of magnetic systems. Therefore, instead of trying to get rid of disorder in experimental systems, people began to find ways of incorporating disorder into theoretical models.

5. As in the case of pure or ideal systems, we fall back on models of magnets to study systems with quenched disorder.

These we call RANDOM MAGNETS.
6) The study of random magnets has been fruitful with and we have been led to a better understanding of
- structural phase transitions and charge density waves in random alloys
- melting of intermetallics
- dirty superconductors
- fluids and superfluids in porous media
- adsorption and wetting on disordered surfaces.

7) As in the case of pure materials, the renormalization group provides the unifying framework for understanding these random systems. The two Chen's are going to discuss this treatment on Thursday.

8) I will briefly discuss three main classes of random magnet models
- Random Exchange Model
- Spin Glasses
- Random field Ising model
9. Pure Magnets: The Ising Model

\[ H = -J \sum_{i,j} S_i S_j \]

**Properties**

- \( H \) has global spin flip symmetry.
- \( kT > J \): Paramagnet
- \( kT \ll J \): Ferromagnet

Order parameter: \( m = \langle S_i \rangle \), the "spontaneous magnetization density". Continuous phase transition at a critical temperature \( T_c \) (of order \( J \)): \( m \) diverges and \( m \) vanishes algebraically.

This intuitive picture is correct for \( d \geq 1 \). But in 1d, thermal fluctuations destroy order at any finite temperature.

\[ d_{c} = 1 \quad \text{for IM} \]

dc: lower critical dimension: The lowest dimension above which an ordered phase can exist at non-zero temperature.
Random Exchange Model

\[ H_{\text{RE}} = \sum_{i,j} J_{ij} S_i S_j \]

\( (J_{ij} > 0 \text{: ferromagnetic coupling}) \)

Properties

For \( d > d_c = 1 \) ferromagnet below \( T_c \).

Early experiments suggested that the transition is broad or smeared.

But the Harris criterion and RG calculations have demonstrated that these systems can have sharp transitions in the absence of macroscopic inhomogeneities or large range correlations in the impurity atoms.

The smeared date later shown to be due to artefacts or macroscopic inhomogeneities.

Conclusion: Equilibrium behavior of random exchange ferromagnets is qualitatively the same as that of pure systems.
\[ H_{\text{eq}} = - \sum_{i,j} J_{ij} S_i S_j \]

\[ (J_{ij} \text{ both } +ve \text{ and } -ve) \]

The ferromagnetic and antiferromagnetic interactions compete to give frustration.

Finding the ground states of these systems becomes a non-trivial problem.

If the density of -ve couplings is small, the ordered phase will be generally like a normal ferromagnet.

If the density of -ve couplings is large, however, the behavior becomes very subtle and the existence and nature of the ordered phase are called into question.
Random-Field Ising Model

\[ \mathcal{H}(\mathbf{S}) = -J \sum_{\langle i,j \rangle} S_i S_j - \sum_i h_i S_i \]

where \( h_i \) is a random variable, chosen from an even distribution, with \( \pm 1 \), or at most short-range correlations between its values at different sites.

This model describes a variety of experimental systems. Example:
\[ \text{Fe}_x \text{Zn}_{1-x} \text{F}_2 \] (a disordered Ising antiferromagnet in a uniform field)

Robin will talk more about such systems.

Properties

Competition: Align ferromagnetically due to \( J \) or be uncorrelated due to \( h_i \).

This model is spin-flip symmetric in a statistical sense: Let \( h_i \rightarrow -h_i \), \( S_i \rightarrow -S_i \)

then \( \langle M \rangle_{\text{av}} \text{ (ensemble)} = 0 \) for any even distribution of \( h_i \).

Order Parameter: \( \langle M \rangle_{\text{av}} \text{ (ensemble)} \)
So, if
$J$ dominates: $\langle m^2 \rangle_0 \neq 0$ (ferromagnetic)
$h_i$ dominate: $\langle m^2 \rangle_0 = 0$ (paramagnetic)

**Expect:** let $h$ be the typical magnitude of $h_i$. Then at low temp,

- $h/J \gg 1$: paramagnet
- $h/J \ll 1$: ferromagnet

When $h/J \gg 1$, it is a paramagnet at all temperatures and any dimension.
But when $h/J \ll 1$, the situation turns out to be more subtle.

One line of argument suggests that $ld = 2$, while a second argument indicates that $ld = 3$.

This becomes a serious point of contention among both theorists and experimentalists, the resolution of which will be presented by Robin in the next talk.

This completes the brief overview of Random Magnets. We now proceed to the second part of the talk on the Harris Criterion.
PART – II : THE HARRIS CRITERION

1. Real systems almost always have impurities. This makes it important to determine if quenched (or frozen-in) disorder affects critical behavior.

2. When we treat the positions of the impurity atoms as fixed and trace over only the magnetic degrees of freedom, we say that the system has quenched disorder.

3. In renormalization group language, the question we are asking is "Is disorder a relevant variable at the critical fixed point of the pure system?"

4. The route to answering this question should involve quantifying the disorder and estimating the corresponding eigenvalue of the linearised RG taking disorder into account.
5. In general, the critical behavior of a disordered system is extremely complicated. In an attempt to begin to address the effects of disorder, Harris presented a heuristic design criterion for when the critical behavior of the disordered system does not differ from that of the pure system.

6. We begin by saying that when the system is pure, it undergoes a continuous phase transition at a critical temperature $T_c$.

\[ T_c: \text{critical temp. of pure system} \]

7. We then introduce disorder and denote the strength of the disorder by $p$. So,

\[ p = \text{mean impurity concentration}. \]

8. In general, the critical temperature of this disordered system will depend on the strength of the disorder. So, say

\[ T_c(p): \text{critical temp. of dis. system} \]
Therefore, the correlation length will go as:

\[ \xi \sim |T - T_c(P)|^{-\nu} \]

The effect of disorder on the pure system may be viewed as changing the local coordination number or exchange interaction. From mean field theory, we know that:

\[ T_c = \frac{2dJ}{k_B} \]

where \( 2d \): coordination number

\( J \): exchange interaction

Therefore, the variation in the coordination number or exchange interaction causes \( T_c \) to vary from point to point in the impure sample. Therefore, \( T_c \) becomes: \( T_c(\tilde{\rho}) \)
11. To model the disorder dependent distribution of $T_c$, we define

$$\delta T_c(n) = n - T_c(p)$$

where $\delta T_c(n)$ is a Gaussian Random Function with mean zero.

12. Consider a region of linear dimension $L \gg a$. The fluctuation in $T_c(p)$ averaged over this region should go as

$$\Delta T_c(p) \sim \frac{1}{\sqrt{V}} \approx L^{-d/2}$$

Think about broken bonds or missing atoms.

13. The Central Argument: Over a correlation volume, the fluctuations in $T_c$ should be small compared with $|T - T_c(p)|$ as $T \rightarrow T_c(p)$ in order that the transition be well defined.
At the fixed point, we have

\[ e \sim |T - T_c(p)|^{-\gamma} \]

So, the criterion for the existence of a well-defined transition becomes

\[ |T - T_c(p)|^{1/2} \ll |T - T_c(p)| \]

as \[ |T - T_c(p)| \to 0 \]

Thus, \( \frac{v_d}{z} > 1 \) or \( v_d > 2 \).

This is the Harris Criterion.

From the hyperscaling relation, we have

\[ 2 - 2 = v_d \]

\( \Rightarrow \boxed{2 < 0} \) Equivalent form of the Harris criterion.
16. So, when the Harris criterion is satisfied, disorder is irrelevant and the system has the same critical behavior with possibly a different $T_c$.

When the Harris criterion is not satisfied, the disordered system has a new fixed point with new critical exponents. These critical exponents then satisfy the Harris criterion. In particular, the $T_c$ is never true at the disordered fixed point.