What Is Temperature?

Closed, insulating box, with small system and large heat bath, total energy $E_1 + E_2 = E$ fixed.

Assume system and heat bath are coupled weakly, so that $E_1$ doesn't depend on the state of the bath and $E_2$ independent of which system state is occupied.

All states of the combined system are equally likely.

→ States of the subsystem at energy $E_1$ will occur with relative probability given by the volume $S_2(E-E_1)$ of the energy surface of the heat bath.

$$P_1(E_1) \propto S_2(E-E_1)$$

Let's expand $$S_2 = k_B \log (RE - \varepsilon)$$

$$P_1(E_1) \propto e^{+ \frac{1}{k_B} S_2(E-E_1)} = e^{\frac{S(E)}{k_B} - E_1 \left( \frac{\partial S}{\partial E} / k_B \right) + ... - \frac{E_1}{k_B T}}$$

where $\frac{1}{T} = \frac{\partial S}{\partial E}$ is the inverse of the temperature.

Entropy cost for stealing energy $\varepsilon$: $-S(E) + \frac{\partial S}{\partial E} \varepsilon$.