Entropy of the Ideal Gas

The state of a classical distribution of particles is a point in phase space

\[(P, X) = (\vec{P}_1, \ldots, \vec{P}_N, \vec{X}_1, \ldots, \vec{X}_N)\quad \text{6N dimensions.}\]

An ideal gas has no potential energy; we thus know the area of the energy surface in phase space

\[S(E) = S_x \cdot S_{\text{TP}}(E) \cdot \left(\frac{1}{\hbar}\right)^{3N}\]

\[= \left(\frac{V}{N!}\right)^{3N} \cdot \frac{(2\pi)^{3N/2}}{(3N/2)!} \cdot \left(\frac{2mE}{\hbar^2}\right)^{3N/2} \cdot \frac{1}{\hbar^3}\]

Identical Particles, Box Size V

Surface area of sphere in momentum space (up to small terms \(\to 0\) as \(N \to \infty\))

\(\Rightarrow\) Quantum mechanics gives "natural unit" of phase-space volume \(= \hbar = 2\pi\hbar\)

multiply \(S(E)\) by \(\left(\frac{1}{2\pi\hbar}\right)^{3N}\). Replace \(N!/(3N/2)!\) by Stirling...\n
\(\text{HW 1.5}\)

\[S(E) = k_B \log S(E)\]

\[S(E) = \frac{N}{2} k_B + N k_B \log \left[\frac{V}{N \hbar^3} \left(\frac{2\pi mE}{3N/2}\right)^{3/2}\right]\]

\(\text{HW 1.4}\)
Pressure on the Ideal Gas

In 1.2.c and 1.4 on the HW, much ado about pressure. Simple calculation...

Pressure \( P = \frac{F}{A} = \frac{dP}{dt} \)

\[
= \left( \frac{\text{# of collisions in } \Delta t}{A} \right) \frac{\text{momentum transferred per collision}}{P_{x}} \cdot \int P(p_{x}) \, dp_{x} \]

\[
= \int \left( \frac{N}{V} \frac{P_{x}}{m} \right) (2P_{x}) \cdot P(p_{x}) \, dp_{x} \]

\[
= \frac{N}{V} \left\langle \frac{P_{x}^{2}}{m} \right\rangle = 2 \left\langle \text{Kinetic Energy} \right\rangle = k_{B} T
\]

\[
P = \frac{Nk_{B}T}{V} \quad \text{[}PV = Nk_{B}T\text{]}
\]

Ideal gas law.

*Why is \( S \) so messy? \( P \) is simple! \( \mu \) is not so simple [HW 1.4.c.e]*