Isothermal Expansion of Ideal Gas

Heat Flow $Q \Rightarrow \frac{\Delta S}{\Delta Q} = \frac{1}{T} \Rightarrow \Delta S = \int \frac{d\theta}{T}$

First Law (Energy Conserved) $\Rightarrow dQ = PdV$

$\Delta S = \int_{V_1}^{V_2} \frac{PdV}{T}$

But $P = \frac{N\langle p^2 \rangle}{m} = \frac{N k_BT}{V}$

From velocity entropy:

$\frac{1}{T} \int dQ = \int_{V_1}^{V_2} \frac{N k_B}{V} dV = Nk \log_e \left( \frac{V_2}{V_1} \right)$

Thermodynamics derives Configurational Entropy formula

(Similar argument HW 1.2, Hard Disks).
"Counting" Entropy and Entropy of Mixing

On Blackboard, which ones are white?

\[ S_0 = \frac{N}{2} k_B \log \left( \frac{V}{N_0} \right) + \frac{N}{2} k_B \log \left( \frac{V}{N_2} \right) \]

- White atoms on one side
- Black atoms on other side

Remove Partition: Mixed Entropy of Integrated System

\[ S_1 = \frac{N}{2} k_B \log \left( \frac{2V}{N} \right) + \frac{N}{2} k_B \log \left( \frac{2V}{N} \right) \]

\[ \Delta S = N k_B \log 2 \quad \text{Entropy of Mixing} \]

- Get \( \log 2 \) every time you place an atom without looking into one of two boxes

\( S = k_B \log \left( \text{# of "different" configurations of system consistent with current knowledge} \right) \)

DO AF KIT

\[ \text{HW 2.1: Entropy of Glasses} \]

- \( S \) measures disorder; lack of knowledge about detailed arrangement of microscopic details
Non-Equilibrium Entropy

What is the entropy of an ensemble that is not in equilibrium?

- Want \( S = k_B \log \mathcal{Z} \) for \( p \) uniform on energy-surface

\[
p = \frac{1}{\mathcal{Z}} \quad S = -k_B \log p = -k_B \int \frac{d\mathcal{Z}}{\mathcal{Z}} \log p = -k_B \int p \log p \, d\mathcal{Z}
\]

Try this as a general formula, good for non-equilibrium \( p(E, Q) \) arbitrary.

**HW 2.2! Shannon Entropy**

- For constant \( E \), \( S \) is maximum for \( p = \frac{1}{\mathcal{Z}} \)

- For constant average \( E \), \( S \) is maximum for \( p \propto e^{-\beta E} \)

- \( S \) is additive. If we have two uncoupled systems \( R \) & \( L \),

\[
S_R = k_B \sum_i p_i^R \log p_i^R \quad S_L = -k_B \sum_i p_i^L \log p_i^L
\]

The probability that the combined system is in the state \([ij]\) is \( P_{ij} = P_i^R P_j^L \)

\[
S = k_B \sum_i \sum_j p_{ij} \log p_{ij} = -k_B \sum_i \sum_j p_i^R \log p_i^R + p_i^L \log p_i^L
\]

\[
= -k_B \left( \sum_i p_i^R \log p_i^R \right) P_j^L + P_i^R \left( \sum_j p_j^L \log p_j^L \right)
\]

\[
=-k_B \left( \sum_i p_i^R \log p_i^R \right) \text{SR} + \sum_j p_j^L \log p_j^L \text{SL} = \text{SR} + \text{SL}
\]
• Does entropy increase?

• Thermodynamics
  Cannot cycles, reversible engines
  Can't make perpetual motion machines
  \[ \rightarrow \text{Entropy increases} \]

• Many approximate theories
  Monte Carlo, Boltzmann equations,
  Diffusion, ...
  \[ \rightarrow \text{Entropy increases} \]

• Simple calculation (HW 2.7)
  \[ \rightarrow \text{Entropy is constant, both} \]
  in classical mechanics and in
  quantum mechanics, for closed
  systems.
  \[ \rightarrow \text{Increase of entropy due} \]
  to coarse-graining: information
  lost in graininess of measurement
  tools, ...