The Einstein Relation

Under a constant force $F$, equilibrium density $p_0(Q) = e^\frac{FQ}{kT}$

$$0 = \frac{dp_0}{dt} = -\frac{m}{F} \nabla p_0 + D \nabla^2 p_0 = \frac{mF^2}{kT} + \frac{DF^2}{(kT)^2}$$

Equilibrium

$$\frac{D}{kT} = \eta$$

Diffusion Viscosity

Einstein Relation

Gravity $\frac{-mgh}{kT}$
Partial Traces & Free Energies

What happened to the water molecules?

We "integrated them away."

\[ Z = \int \text{d}Q_{\text{pollen}} \int \text{d}q_1 \ldots \int \text{d}q_N e^{-\beta V(Q_{\text{pollen}}, q_1, \ldots, q_N)} \]

\[ = \int \text{d}Q_{\text{pollen}} \tilde{Z}(Q_{\text{pollen}}) \text{ Integrating over Unimportant Degrees of Freedom} \]

\[ F(Q_{\text{pollen}}) = -kT \log \tilde{Z}(Q_{\text{pollen}}) \]

\[ \rho(Q_{\text{pollen}}) = \frac{\tilde{Z}(Q_{\text{pollen}})}{Z} e^{-\beta F(Q_{\text{pollen}})} \]

A(T, V, N) \rightarrow F(Q, T, V, N)

Stat Mech Still Applies: F replaces H

Can be non-trivial! Pollen grain, water molecules as hard spheres, next to a rigid wall

→ effective attraction due to overlapping excluded regions

\[ \tilde{F}(Q) \rightarrow -V_{\text{pollen}} \]