Coarsening

Oil & vinegar = Salad Dressing

Shake \Rightarrow Mixes
Wait \Rightarrow Separates
Short times \Rightarrow Gravity not important

Rocks

Granite
Lava, Molten Rock Dissolved Together
Cool \Rightarrow Separates into Different Phases
Cool fast \Rightarrow Fine-Grained Pumice (Gas Bubbles)
Cool Slowly \Rightarrow Granite, Quartz, Copper Crystals, Crystals

Single Phase: Polycrystal Grain Size Coarsening

Ising Model

$$T>T_c \Rightarrow Mixed$$
$$H=0, \quad High T \Rightarrow Below T_c$$

\Rightarrow Up spin regions separate from down spin regions

Some systems conserved order parameter, Ising & grain growth non-conserved,
Rough Arguments for Coarsening

(Dimensional Analysis, Caricature \( \rightarrow \) Works Brilliantly)

- Sphere of Radius \( R \)
- Feature of length \( R \)

How long until features of scale \( R \) evaporate?

- Unlike nucleation, \( \Delta \mu \approx 0 \)
  - Using \( T = 0, \mu_T = \mu_L \)
  - Liquid-gas, fixed volume
  - Conservation order parameter \( \rightarrow \) Pressure rises until coexistence \( \Delta \mu = 0 \)

- Energy \( \approx 4\pi R^2 \sigma \)

- Inward force \( \frac{\text{Area}}{\text{Difference}} \) = Pressure

  \[
  P_{in} = \frac{\Delta \text{Energy}}{2R} = \frac{8\pi\sigma R}{4\pi R^2} = \frac{2\sigma}{R}
  \]

  "Generally true if \( R = \) Mean curvature at surface"
Non-Conserved Order Parameter

Interface Mobility

\[ \hat{\mathbf{v}} = (\mu \mathbf{P}) \hat{n} \]

(Derive from Bloch wall, equation of motion \( \frac{\partial M}{\partial t} = -\gamma \frac{\partial E}{\partial M} \ldots \))

\[ \frac{dR}{dt} = V = \mu \left( \frac{-2\sigma}{R} \right) \int_{R_0}^{R_f} dR = \int_{0}^{t_f} -2\sigma \mu \, dt \]

\[ \frac{R_0^2}{2} = 2\sigma \mu t_f \quad t_f = \frac{R_0^2}{4\sigma \mu} \]

Characteristic size of smallest features

\[ L(t) \sim \sqrt{t} \sim t^{1/2} \]

Power-law coarsening length

\[ L \sim t^{1/2} \text{ Non-Conserved} \]
Conserved Order Parameter

\[ \Delta \mu \approx \frac{2D_0}{R_L} \]

Energy gain per particle in (Pressure) (Charge per Particle)

\[ \text{Current } J \approx \frac{D}{R} \int \frac{\Delta n}{\partial R} \text{ m Force} \frac{2D_0}{R^2} \text{ m Diffusion Constant} \]

\[ V_{\text{drop}} = \frac{4}{3} \pi R_{\text{drop}}^3 \]

\[ \frac{dV_{\text{drop}}}{dt} = J \cdot A_{\text{drop}} \approx \left( \frac{D_0}{R^2} \right) \left( R^2 \right) \approx D_0 \]

\[ \frac{dR^3}{dt} = 3R^2 \frac{dR}{dt} = -D_0 \]

\[ \int_{R_0}^{R} R^2 dR = \int_{0}^{t} -D_0 p \, dt \quad R_{\text{final}}^3 = v t \]

Conserved Order Parameter \( L \approx v t^{1/3} \)
• Other phase transitions nucleated:

- Liquid–Solid

- Crystalline anisotropy \( O(\lambda) \)
  
  (high symmetry close-packed surfaces lower surface tension)

- Critical droplet not sphere
  Equilibrium Crystal Shape

• Growth Dynamics
  Latent heat, impurities must be driven away

- Instability to Dendrites

Snowflake
Large Solid Block Under Load, T > 0

What is the Ground State?

Stretched: Energy & Volume

Broken: Energy & Surface Area

Alex Buechel: Rate of Fracture due to Thermally Activated Critical Cracks \( \Gamma(P) \)

(Solid \rightarrow Gas) transformation (in \( CO_2 = Dry Ice \)) Mediated by Crack Formation \( \Gamma \rightarrow -P \)

- Metastable Phase formally has complex free energy \( \mu \rightarrow \mu + ic\Gamma \)

- Essential Singularity in imaginary part \( \Gamma''(P) \)

\[
\Gamma''(P) = \begin{cases} 
0 & P > 0 \\
\frac{A}{e^{P_0^2}} & P < 0
\end{cases}
\]

- Kramers-Krönig \( \Rightarrow \text{Re}(\mu) \) also singular at \( P = P_v \approx 0 \)

- Nonlinear elastic constants = Expansion about \( P = 0 \) \( \Rightarrow \) Elastic theory has Zero Radius of Convergence,