Pre-class Preparation

Wednesday
Read: Chapter 7, Sec. (7.7) (Metals and the Fermi gas)

1. Is sound a quasiparticle?②

Sound waves in the harmonic approximation are non-interacting – a general solution is given by a linear combination of the individual frequency modes. Landau’s Fermi liquid theory (footnote (23), page 144) describes how the non-interacting electron approximation can be effective even though electrons are strongly coupled to one another. The quasiparticles are electrons with a screening cloud; they develop long lifetimes near the Fermi energy; they are described as poles of Greens functions.

(a) Do phonons have lifetimes? Do their lifetimes get long as the frequency goes to zero? (Look up ultrasonic attenuation and Goldstone’s theorem.)

(b) Are they described as poles of a Green’s function? (We will cover this in Chapter 10. Browse in particular Exercise (10.9), ‘Quasiparticle poles and Goldstone’s theorem’.)

(c) Can you think of an analogy to a screening cloud?

Friday
Read: Chapter 8, Sec. (8.1) (The Ising model)

2. Solving the Ising model with parallel updates.②

Our description of the heat-bath and Metropolis algorithm equilibrates one spin at a time. This is inefficient even in Fortran and C++ (memory nonlocality); in interpreted languages like Python or Mathematica conditional loops like this are really slow.

Could we update all the spins at once, thermalizing them according to their current neighborhood? If not, can you figure out a way of bringing many of the spins to equilibrium with their neighborhoods in one set of array operations (without ‘looping’ over spins)? (No implementation is needed – just a description of the method.)

Monday
Read: Chapter 8, Sec. (8.2) (Markov Chains)
3. **Coin flips and Markov.** (Mathematics) ②

A physicist, testing the laws of chance, flips a coin repeatedly until it lands tails.

(a) **Treat the two states of the physicist** (‘still flipping’ and ‘done’) as states in a Markov chain. The current probability vector then is
\[ \vec{\rho} = \begin{pmatrix} \rho_{\text{flipping}} \\ \rho_{\text{done}} \end{pmatrix}. \]

Write the transition matrix \( P \), giving the time evolution
\[ P \cdot \vec{\rho}_n = \vec{\rho}_{n+1}, \]
assuming that the coin is fair.

(b) **Find the eigenvalues and right eigenvectors of** \( P \). Which eigenvector is the steady state \( \rho^* \)? Call the other eigenvector \( \tilde{\rho} \). For convenience, normalize \( \tilde{\rho} \) so that its first component equals one.

(c) **Assume an arbitrary initial state is written** \( \rho_0 = A \rho^* + B \tilde{\rho} \). What are the conditions on \( A \) and \( B \) needed to make \( \rho_0 \) a valid probability distribution? Write \( \rho_n \) as a function of \( A, B, \rho^*, \) and \( \tilde{\rho} \).

**Exercises**

7.14: Bose condensation: the experiment

7.15: The photon-dominated universe

7.16: White dwarfs, neutron stars, and black holes

8.2: Ising fluctuations and susceptibilities. Use Matt Bierbaum’s simulation at [http://mattbierbaum.github.io/isin.js](http://mattbierbaum.github.io/isin.js)