Sound and Atoms: The One Dimensional Crystal

- Atom Mass $M$
- Chemical Bond = Spring Constant $K$
- Chain of Balls and Springs,
- Longitudinal Wave:
  - $u_{n+1}, u_n, u_{n+1}, u_{n+2}, u_{n+3}$
  - $\delta x = \text{Spring Length}$
  - $n\delta x = \text{Undeformed Position}$
- $x_n = n\delta x + u_n = \text{Current position}$

Bond B is stretched: length is $\delta x + u_{n+1} - u_n$
- Force on atom $n$ is $K(u_{n+1} - u_n)$
  - to right

Bond A is squeezed: length is $\delta x + u_n - u_{n-1}$
- If $u_n - u_{n-1} < 0$
- Force on $u_n > 0$
  - Force on $u_n > 0$
- $\Rightarrow$ Note

$m\frac{d^2 u_n}{dt^2} = m\frac{d^2 u_n}{dt^2} = K(u_{n+1} - u_n) - K(u_n - u_{n-1})$
- $d^2u_n/dt^2 = \frac{K}{m} (u_{n+1} - 2u_n + u_{n-1})$
- Should look familiar?

Real chain of atoms $\Rightarrow$ Approximate wave equation
Sound Demo

Need:
- Mac, running MacCRO (Cathode Ray Oscilloscope?)
- Tuning Fork, medium
- Organ Pipe, medium
- Cardboard tube “Pipette”
- Sonometer (string)

Input Settings: Set Gain 110

Voice:
- Oscilloscope:
  - Time Scale 10 ms/div
  - Trace A 50 mV/div
  - Triggering
  - Deep voice: hum, show periodicity in waveform

Tuning Fork (medium):
- Time Scale 0.1 ms/div
- Trace A 10 mV/div
- Show Sinusoidal waveform

Spectrum Analyzer:
- Resolution 2 Hz
- 0-2000
- 0-5
- Label Peaks
- See single harmonic

Voice:
- 0-500
- 0-5
- Freeze Display
- See all the harmonics

Medium Pipe Organ (ask for volunteers):
- 0-2000
- 0-5
- See all harmonics

Pipette:
- 0-1000
- 0-5
- See odd peaks high

Sonometer:
- 0-500
- 0-2
- Pluck middle, odd harmonics (node at center for even harmonics)
- Pluck near end, all harmonics
SOUND

DEMO: ORGAN PIPES
OSCILLOSCOPE & MIKE
TUNING FORK
FREQUENCY ANALYZER

Shape of wave
Harmonics

ORGAN PIPE, FLUTE, TUBA, ...

Attn Displacement $s(x)$

$\uparrow$ Displacement zero

Pressure $P$
Pressure Matches Outside
(Density $\rho$)

Pressure, Density Fluctuate About
Atmospheric $P_0$

Fixed Boundary

Free Boundary (Approximately)
Wave Equation for Sound in One Dimension

Air, water, solids: Pressure depends on Volume

\[ P = P_0 - B \left( \frac{\Delta V}{V} \right) \]

\[ B = \text{Bulle Modulus} \]

\[ \text{Good for small } \frac{\Delta V}{V} \]

Area

\[ A \]

\[ S(x) \rightarrow S(x + \delta x) \]

\[ V = A S_x \]

\[ V + \Delta V = A \left\{ S_x + S(x + \delta x) - S(x) \right\} \]

\[ \Delta V = A \left[ S(x + \delta x) - S(x) \right] \]

\[ P - P_0 = -B \frac{\Delta V}{V} = -B \frac{A \left[ S(x + \delta x) - S(x) \right]}{A S_x} = -B \frac{\partial S}{\partial x} \]

Pressure is Force per unit Area

Force = \[ A \cdot P(x) - A \cdot P(x + \delta x) \]

\[ = MA \]

\[ \rho A \delta x \frac{\partial^2 S}{\partial t^2} \]

\[ \rho A \delta x \frac{\partial^2 S}{\partial t^2} = A^2 (P(x) - P(x + \delta x)) \]

\[ -B \frac{\partial S}{\partial x} \]

\[ \frac{\partial^2 S}{\partial t^2} = \frac{1}{\rho} \frac{P(x) - P(x + \delta x)}{S_x} = -\frac{1}{\rho} \frac{2P}{\partial x} = \frac{B}{\rho} \frac{\partial^2 S}{\partial x^2} \]
Wave Equation for Sound

\[
\frac{\partial^2 s}{\partial x^2} = \frac{\beta}{\rho} \frac{\partial^2 s}{\partial t^2}
\]

Velocity of Sound

in Air, 20°C

\[= 343 \text{ m/s} \sim 0.5 \text{ mile/s} \]

\[s = \text{Sinusoidal}\]

What's the Pressure for Traveling Wave?

\[s(x,t) = S_{\text{max}} \cos \left( \frac{2\pi x}{\lambda} - 2\pi ft \right)\]

\[p - p_0 = -\beta \frac{\partial s}{\partial x} = \frac{2\pi \beta}{\lambda} S_{\text{max}} \sin \left( \frac{2\pi x}{\lambda} - 2\pi ft \right)\]

\[p_{\text{max}}\]

What's the Kinetic Energy Density?

\[
\text{Kinetic Energy} = \frac{1}{2} \left( \frac{M}{V} \right) \left( \frac{\partial s}{\partial t} \right)^2 = \frac{1}{2} \rho \left( \frac{2s}{3t} \right)^2
\]

Potential Energy = Kinetic Energy for Traveling Wave

Total Energy Density = \[p \left( \frac{2s}{3t} \right)^2\]

\[= \rho S_{\text{max}}^2 \left( \frac{2s}{3t} \right)^2 \sin^2 \left( \frac{2\pi x}{\lambda} - 2\pi ft \right)\]
What's the **Intensity** of a traveling sound wave?

Intensity = \( \frac{\text{Power}}{\text{Area}} \) = \( \frac{\text{Energy Density}}{} \times \text{Velocity} \)

\[
I = \sqrt{\rho p} \left( \frac{\partial \psi}{\partial t} \right) = \sqrt{\rho p} \left( 2\pi \ell \right) \frac{s}{\text{max}} \frac{\sin^2 \left( \frac{2\pi x}{\lambda} - 2\pi ft \right)}{\frac{2\pi x}{\lambda}}
\]

What's the **Average Intensity**?

*Useful Trick*: Average of \( \sin^2 \) is \( \frac{1}{2} \)

\[
\sin^2 + \cos^2 = 1
\]

average \( \sin^2 = \frac{1}{2} \)

average \( \cos^2 = \frac{1}{2} \)...

\[
\text{Average Intensity} = \sqrt{\rho p} \left( 2\pi \ell \right)^2 \frac{s}{\text{max}} \frac{1}{2}
\]

[Express in terms of \( P_{\text{max}}^2 \)]

**Units**: Intensity = Joules/sec per unit area = Watts/m\(^2\)

At 1000 Hz, you can hear \( I_0 = 10^{-12} \) W/m\(^2\) = 1 dB

Corresponding to air displacing \( s_{\text{max}} = 10^{-11} \) m \( \sim \frac{1}{30} \) atoms

A power mower \( I = 10^{-2} \) W/m\(^2\) = ten Giga (\( I_0 \))? [Use Log Scale]

Decibels = \( \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \)

Lawn Mower = \( 10^{10} \) \( I_0 = 100 \) dB