Prelim I Answer Key
Physics 218: Waves and Thermodynamics
Fall 2001, James P. Sethna

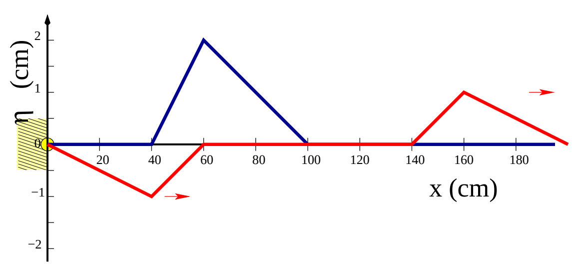
The mean for the class was 76; the standard deviation was 19. Five people got perfect scores: three would have done so even if I didn't mess up problem M5...

Multiple Choice (10 points each)

M1. D.
M2. D. The answer E is not correct: the pressure is fixed at atmospheric $p = 0$ at the open end: the closed end acts as a free boundary condition for the pressure. This can be deduced, if you don't remember it, from $\partial p/\partial x = -\rho \partial^2 s/\partial t^2$ on the formula sheet: since s is fixed at that end its acceleration vanishes, so p has free boundary conditions.
M3. B.
M4. C.
M5. A. I had some errors in the first version of this problem, but they've been corrected in the current version.

Short Answer.

S1(A). Kinetic energy = 0 ergs (5 points), Potential energy = 1.2 ergs (10 points). I gave partial credit of 3 points for showing that you knew the formula for potential energy, two points for knowing that you had to integrate it, and two points for getting $\partial \eta/\partial x$ correct from the graph.
S1(B) (15 points). The triangle splits into two half-pulses, and the leftward-moving one reflects at the fixed boundary, inverting and flipping:



I gave three points of partial credit for getting $c = 2$cm/s correct, two points for writing that $ct = 100$cm, two more points for showing any packet moved by 100cm, two points for splitting the pulse into two half-sized pulses, and two points if you correctly invert and flip the reflected pulse.
S1(C) (20 points). The solution

$$\eta(x,t) = 1/2\left(f(x-ct) + f(x+ct) - f(-(x-ct))\right).$$

The first term is the right-moving pulse, the second term is the left-moving one, and the third one is the fictional pulse coming in from the wall which becomes the reflected pulse.

You can verify that (a) at $t = 0$ it satisfies the initial condition $\eta(x, 0) = 0$ and the initial velocity condition $\partial\eta(x, t)/\partial t = 0$ for $x > 0$ (the fictional pulse is still behind the wall), (b) it satisfies the boundary condition $\eta(0, t) = 0$ for $t > 0$ (at negative times the rightward pulse would hit the wall), and (c) it's a superposition of solutions to the wave equation.*

I gave partial credit of ten points for getting two pulses correct (several people forgot the reflected pulse), five points if you wrote two pulses but forgot the 1/2, two points if you mentioned the third pulse (but couldn't figure out how to implement it), three points for getting the third pulse but in the wrong position.

I also gave partial credit to those who tried to write a Fourier series solution. It really should be a Fourier transform, since the string extends to infinity, so the answer really is an integral rather than a sum. In principle you could rearrange terms to get the answer above, but I would have given complete credit if you found the Fourier coefficients, even if you left the answer as an integral. I gave partial credit of five points for recognizing that the answer could be written as a sum over cosines and sines, two points for recognizing that the terms involving $\sin(\omega t)$ had to vanish (since $\partial\eta/\partial t = 0$ at $t = 0$), two points for recognizing that terms involving $\cos(kx)$ had to vanish (due to the fixed boundary at $x = 0$), and two points for explicitly stating that you could in principle get the coefficients in the expansion from the Fourier series formulas (if you had another hour or two to calculate stuff).

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* An even more elegant solution is a superposition of four pulses:

$$\eta(x, t) = 1/2\,(f(x - ct) + f(x + ct) - f(-(x - ct)) - f(-(x + ct)))$$

this one also works for negative x and negative t.