Statistical Mechanics: 
Probability $\rho(S)$ to be in state $S$. 

Ising model: $S = \{S_i\}$,  
Sites $i$ on a (square) lattice $i = (x, y)$,  
Spins $S_i = \pm 1$. 

Equilibrium statistical mechanics: 
Energy $E(S)$,  
Boltzmann probability distribution  
$\rho(S) \propto \exp(-E(S)/k_B T)$.  

High temperatures: 
States have equal weight  
Low temperatures:  
Low-energy states predominate 

Ising model:  
$E(S) = -\sum_{\langle i, j \rangle} J S_i S_j$,  
Lowest energy state ($J > 0$), all spins up (+1) or down (-1)  
*Broken symmetry* ferromagnetic phase  
High temperature, *paramagnetic* phase  
Transition at $T_c$: fluctuations
Markov chain:
  Dynamics for statistical models
  Transition rate $P_{S'S}$ from $S$ to $S'$
  Markovian: independent of history

Markov chain properties:
  Detailed Balance:
  Equilibrium flux $S \rightarrow S' = \text{flux } S' \rightarrow S$
  $P_{S'S} \cdot \rho(S) = P_{SS'} \cdot \rho(S') =$
  Ergodic: Every state can be reached

A Markovian model that is ergodic and satisfies detailed balance will eventually approach equilibrium.

Ising model dynamics:
  Heat bath:
  Pick a spin at random, measure flip $\Delta E$
  Equilibrate it to it’s current environment
  Metropolis
  Pick a spin at random, measure flip $\Delta E$
  If $\Delta E < 0$, flip down
  If $\Delta E > 0$, some chance to flip up
  Wolff algorithm
  Clever generation of cluster flips
  Vastly faster dynamics near $T_c$
  Satisfies detailed balance, plus magic
Continuous-time; Bortz/Kalos/Lebowitz
Keep lists of spins in different environments
Calculate total rate to flip
Find which spin environment flips next
Flip random spin in that environment
Vastly faster at cold temperatures
Preserves dynamics (coarsening)