Bifurcations & Chaos in Iterated Maps I: Chaos & Lyapunov Exponents / Invariant Measure

Myers/Sethna: Computational Methods for Nonlinear Systems

Logistic map:
\[ x_{n+1} = 4\mu x_n (1-x_n) \]
IterateLogistic and the args tuple

\[ x_{n+1} = 4\mu x_n (1-x_n) \]

one-dimensional map (x) with one parameter (\(\mu\))

def Iterate(g, x0, N, args=()):
    """
    Iterate the function g N times, starting at x0
    with extra parameters passed in as a tuple args.
    Return g(g(...(g(x))...)). Used to find a point
    on the attractor starting from some arbitrary
    point x0.
    """

Using scipy convention of an “args tuple” to pass in an arbitrary
number of extra arguments to a generic function:

e.g., scipy.integrate.odeint(dydt, y0, times, args=(), ...)
dydt is a function of y and t with optional additional arguments
dydt(y, t, a,b,c) \Rightarrow scipy.integrate.odeint(dydt, y0, times, args=(a,b,c))

f(x, \mu): f is a function of x with additional arguments args=(\mu,)
Lyapunov Exponent
rate of divergence (or convergence) of nearby trajectories

$$\Delta x_{n+1} \sim \exp(\lambda t) \rightarrow \text{sensitive dependence on initial conditions}$$
Invariant Measure
stationary probability density in chaotic regime

$$\Delta y = (\text{local slope}) \times \Delta x$$

- trajectories get expanded or compressed depending on value of local slope
- at the critical point of the map (at $x=0.5$), slope $\to 0 \Rightarrow$ compression $\to \infty$
- singularities in invariant measure