

A history of the I - V characteristic of CDW conductors

R.E. Thorne¹

¹ *Physics Department, Cornell University, Ithaca, NY 14853, USA*

Abstract. The humble current-voltage (I - V) measurement has proven to be an extremely powerful probe of the physics of condensed matter systems from bulk semiconductors and superconductors to quantum dots and nanotubes. However, doing these deceptively simple measurements “right” so as to unambiguously extract particular bits of physics is hard. This is especially true of charge and spin density wave (CDW and SDW) conductors and related collective transport systems, in part because of the tremendous richness of their physics. The central role of the CDW I - V characteristic in experiment has made it the focus of extensive theoretical study, and the ideas developed have spread to many areas of condensed matter physics. In this short review I discuss some highlights of the study of the CDW I - V characteristic. After nearly 30 years, the fundamental experimental features are finally becoming clear, and they are not accounted for by existing theories.

1. THE BEGINNING: NONLINEAR CONDUCTION, COLLECTIVE PINNING AND ZENER TUNNELING

In 1976, Monceau, Ong, and collaborators [1] made the first observation of nonlinear electrical conduction in NbSe₃. The nonlinearity’s small characteristic electric field scale — on the order of 0.1 V/cm at $T = 50$ K — suggested a characteristic energy scale per electron (roughly eEl , where l is a mean free path) orders of magnitude smaller than $k_B T$. This provided the first clear evidence for collective charge transport by a sliding Fröhlich mode. Fleming and Grimes [2] found that conduction was linear below a threshold electric field E_T , and that nonlinear conduction above E_T was accompanied by coherent oscillations at a frequency proportional to the excess current, clinching the case for sliding CDW transport. The collective pinning theory of Fukuyama, Lee and Rice [4] was confirmed by Ong, Brill and coworkers [3], who showed that E_T in NbSe₃ increased in proportion to the square of the concentration of Ta impurities.

Monceau *et al.* [1] fit their data by a Zener tunneling form $G_{\text{CDW}}(E) = G_\infty \exp(-E_0/E)$. Motivated by this form, Maki [5] analyzed a model of tunneling by soliton-antisoliton pairs. The fit parameters were unreasonable and thermally created solitons seemed likely to dominate transport. Bardeen [6] proposed an alternative model in which tunneling occurred across a small gap determined by the collective pinning, although the precise nature of the tunneling event was never described. The Zener form does in fact provide an excellent fit to experiments on the $T_{P1} = 145$ K CDW in NbSe₃ between $T = 70$ and 130 K [7]. But those of us who supported Bardeen’s interpretation overlooked an obvious problem: the small value of E_0 . For E/E_T between ~ 2 and 200, where the fit works well, $E/E_0 \sim 1 \rightarrow 100$ and $\exp(-E_0/E)$ varies between ~ 0.3 and ~ 1 . Consequently, the fit works only where the “tunneling probability” is close to one and the field dependence is weak, and fails at smaller $\exp(-E_0/E)$ where a tunneling fit would be more likely to apply.

2. ASYMPTOTIC BEHAVIOR AS $E \rightarrow \infty$

In 1982, Sneddon, Cross and Fisher [8] made one of the most robust and directly testable predictions of FLR-based classical elastic models. To leading order in perturbation theory, the conductance at high fields varies as $G_{\text{CDW}}(E) = G_\infty - CE^{-1/2}$. Experiments up to $E/E_T \sim 200$ in high-quality NbSe₃ crystals [7] showed that $G_{\text{CDW}}(E) = G_\infty - C/E$, the behavior expected for the Zener form. This discrepancy

between Ref. [8] and experiment has never been explained, and comparable experiments have yet to be performed in the fully gapped CDW materials. A leading correction proportional to $1/E$ is expected within the model of Ref. 8 in 2D [9], and the static collective pinning in undoped NbSe₃ is 2D [10]. However, the dynamic correlation length shrinks with increasing velocity [9]. At high fields the dynamics should revert to 3D, but calculations of dynamic dimensional crossovers have not been reported.

3. ASYMPTOTIC BEHAVIOR AS $E \rightarrow E_T$

In 1983, Daniel Fisher [11] showed that CDW depinning within a classical elastic model is a dynamic critical phenomenon, in which a reduced electric field $f = (E - E_T)/E_T$ plays the role of the reduced temperature in conventional critical phenomena. The sliding state is characterized by a dynamic length scale that diverges as the depinning transition at E_T is approached from above. The CDW velocity has the scaling form $v(f) \propto f^\zeta$, and in mean field theory $\zeta = 3/2$. Fisher's ideas have stimulated an enormous body of theoretical and computational work in condensed matter physics and nonlinear dynamics, and Ref. 11 is the most cited research article in the field.

So how do the predictions of Ref. 11 compare with experiment? Early measurements yielded $\zeta = 1.3 - 2.3$ in TaS₃ and 1.5 in NbSe₃ [12]. The first experiment to focus on critical behavior gave $\zeta > 2$ in TaS₃ [13]. After it was pointed out that the mean field exponent is an upper bound, new experiments gave $\zeta = 1.24$ for $0.2 \leq f \leq 0.9$ [13].

Many extraneous factors can lead to a smoothing/rounding of the depinning at E_T and an increase of the apparent exponent ζ , and these correlate with a broadening of the spectral width of the coherent oscillations [14]. In the absence of a rigorously valid theoretical prediction for this spectral width (and thus for the elasticity/coherence of the CDW response), the "intrinsic" spectral width is bounded from above by the best spectral width achieved in any experiment. In NbSe₃ at $T \sim 115$ K, this bound is less than 15 kHz for oscillation frequencies between 1 MHz and 200 MHz [15], corresponding roughly to $0.5 < f < 25$, indicating a highly coherent and largely elastic bulk CDW response.

Experiments on these most coherent NbSe₃ samples yielded $\zeta = 1.09$ for $0.05 < f < 10$ [7]. However, the critical regime in ordinary phase transitions is rarely observable beyond reduced temperatures of 0.1, and there is no reason to expect a larger critical regime for dynamic transitions. For $f < 0.1$, the most coherent NbSe₃ samples show complex, sample-dependent structure with a non-monotonic dV/dI , and no evidence whatsoever of scaling behavior [15].

Within the context of the classical elastic model of Fisher [11] and related models, the theoretical picture was clarified by Narayan [16], Middleton [17] and Myers [18]. Narayan's RG calculation in $4-\varepsilon$ dimensions gave $\zeta = 1 - \varepsilon/6$, or $5/6$, $2/3$ and $1/2$ in $D=3$, 2 and 1, respectively. These values have not been observed in any experimental system. The collective pinning lengths in NbSe₃ are micrometers, so that typical ribbon-like samples have dimensions of only $\sim 1 \times 100 \times 100$ static correlation lengths. Numerical simulations by Myers and Middleton suggested that in such samples finite size effects become important before the critical regime is reached, and that few degree-of-freedom dynamics that depend on the details of the disorder may lead to abrupt jumps in $v(f)$ at small f , similar to what is observed in experiment [18]. Perhaps future experiments will find convincing evidence for the dynamic critical behavior that has for so long dominated the broader physics community's conception of what is interesting about sliding CDWs.

4. INTRINSIC PLASTICITY AND CREEP

Motivated in part by experiments on vortex lattices showing complex plastic flows and thermally-assisted creep, and in part by experiments on CDW systems suggesting an incoherent response near the depinning transition, over the last dozen years theorists have attempted to extend elastic models to include plasticity and thermal fluctuations [19,20]. Typical calculations predict that CDW motion begins with incoherent

creep and plastic flow, and that motion becomes more elastic at high velocities. However, the plasticity observed in experiments is due primarily if not entirely to phase slip near current contacts, to shear caused by thickness steps, and to inhomogeneous current injection from side contacts [14]. There is no evidence for bulk plasticity leading to an incoherent CDW response above the collective pinning threshold E_T in any CDW system.

Similarly, incoherent thermal creep below the nominal E_T in bulk CDW systems is irrelevantly tiny, because unlike in vortex lattices, collective pinning energies are orders of magnitude larger than $k_B T$. Incoherent CDW creep has not been observed except in very thin crystals very close to T_P [21].

5. THE ELEPHANT UNDER THE CARPET

In all of the models discussed above, the CDW depins smoothly at E_T and its velocity increases smoothly above it. Yet this prediction of mostly *zero-temperature* calculations only resembles observations at *high* temperatures (above $\sim 2T_P/3$). In all sliding CDW (and SDW) materials discovered so far, below $\sim 2T_P/3$ the I - V relation has a fundamentally different form. This has been evident to experimentalists since before the work of Sneddon, Cross and Fisher, but has largely been ignored by theorists. In the last few years the full scope of the failure of theory has become clear.

In 1982 [22] (and likely well before then) it was noted that the I - V characteristic of the $T_{P2} = 59$ K CDW in NbSe₃ changes dramatically below $T \approx 40$ K. The smooth depinning at high temperatures is replaced by an abrupt and often hysteretic “switch” from zero to finite current. The detailed behavior is highly sample dependent: in some samples only an abrupt drop in dV/dI is observed, and in others dV/dI shows a steep but finite decrease. In all cases the sharp change occurs at fields much larger than E_T measured in the high-temperature regime.

In 1986, Fleming *et al.* [23] reported that in K_{0.3}MoO₃, o-TaS₃, and (TaSe₄)₂I, the CDW conductance above E_T scales with the single particle conductance. Since the latter is due to thermally-activated quasiparticles in these fully gapped materials, both freeze out with decreasing temperature. Mihaly *et al.* [24] showed that at helium temperature this freeze-out leads to essentially insulating behavior up to a relatively large field, at which an abrupt turn-on of CDW conduction causes the total conductance to rise by many orders of magnitude and toward values comparable to those observed at high temperatures.

Itkis and Nad [25] clarified the experimental situation in the fully gapped materials in 1991, pointing out that at lower temperatures two characteristic fields E_T and E_T^* coexist. CDW conduction above E_T freezes out and E_T decreases (as expected due to CDW stiffening) with decreasing T . The abrupt “switching” to a high-conductance state develops on cooling at a much larger E_T^* .

To explain the results of Fleming *et al.* and Mihaly *et al.*, Littlewood [26] proposed a simple model in which descreening of CDW polarizations due to single-particle freeze-out leads to enhanced CDW dissipation and a scaling of the collective and single particle conductances. Littlewood suggested that the CDW-single particle interaction could explain “switching” from a pinned state to high-velocity sliding, and simulations of a similar model by Levy and Sherwin [27] did in fact show switching. However, both models predicted only a single characteristic field rather than the two observed in experiment, and a magnitude for the switching field/velocity that was in error by several orders of magnitude.

In partially-gapped NbSe₃, the situation was confused by a series of experiments in the mid-1980’s by the Berkeley group [28]. These experiments seemed to indicate that “switching” was due to localized macroscopic defects that split the sample into macroscopic domains with different velocities. This confusion was resolved by Adelman *et al.* [29] in 1991, who showed that switching was in fact a bulk phenomenon, just as in the fully-gapped materials. Both the switching threshold E_T^* and the high-temperature threshold E_T scaled inversely with crystal thickness. Both coexisted below $2T_{P2}/3$, with CDW conduction between E_T and E_T^* freezing out with decreasing T , just as in the fully gapped materials. But unlike in the fully-gapped materials, the single particle density remains large below T_{P2} in NbSe₃ and the single particle conductance *increases* with decreasing temperature. The natural conclusion was

that strikingly similar phenomena observed in NbSe₃ and the fully gapped materials have a common origin, and that single-particle descreening is not the primary cause.

In 1999, Serge Lemay [30] made the most remarkable discovery since the early observations of depinning and coherent voltage oscillations. Below $T = 35$ K in NbSe₃ crystals with highly homogeneous currents, he observed coherent voltage oscillations between E_T and E_T^* . Their amplitude and relation to the CDW current density implied that they were the usual “narrow-band noise” associated with collective CDW motion, but their frequency froze out with decreasing temperature. The CDW velocity-field relation determined by measuring this frequency was well described by a modified Anderson-Kim form for thermal creep, $v(E, T) = \sigma_0(E - E_T) \exp[-T_0/T] \exp[\alpha(E - E_T)/T]$. Just above E_T , CDW velocities as slow as cm/year were measured. Yet E_T remained a true threshold, with the velocity dropping by at least five orders of magnitude just below it [31]. This observation of spectacularly slow, creep-like motion with a finite threshold in a regime of highly coherent oscillations implied that CDW motion between E_T and E_T^* is largely elastic and collective, ruling out all models of plastic transport and conventional thermal creep. Measurements on K_{0.3}MoO₃ [32] and o-TaS₃ [33] have been also fit using Eq. 1.

Most recently, Cicak [34] has shown that data sets from seven Ta and Ti doped NbSe₃ crystals can be collapsed onto the fit of Eq. 1 over nearly six orders of magnitude in velocity, yielding consistent values for the barrier $T_0 \approx 540$ K, roughly $0.65 \times 2\Delta$. “Switching” appears to occur not at a characteristic field E_T^* but at a characteristic velocity v^* that is temperature activated.

6. THE I-V CHARACTERISTIC OF CDW CONDUCTORS

From the work described above, we can piece together a consistent experimental description of the I - V characteristic of both partially and fully gapped CDW conductors. At temperatures below roughly $2T_P/3$, the CDW remains pinned and thermal creep is negligible up to the collective pinning threshold E_T . Above E_T , the CDW depins and moves collectively with excellent coherence, as indicated by the small spectral width of the coherent voltage oscillations. But its motion shows simple creep-like temperature and field dependencies. The barrier to creep is comparable to the CDW gap, indicating that it is of local rather than collective origin. This proximity of the creep barrier to the single particle gap produces the rough scaling of single particle and collective conductances in fully gapped materials. At a critical velocity v^* (at E_T^*) that is also temperature activated, a first-order-like dynamical transition occurs to a high-velocity sliding state where activation over local barriers no longer limits motion, and where the conductance is comparable to that observed at high temperatures.

As the temperature increases, local barriers are more easily overcome and the conductance between E_T and E_T^* increases. Above a critical temperature $T_c \sim 2T_P/3$, local barriers no longer limit motion and the first-order transition disappears. Thermal creep is still negligible below E_T , and above E_T the CDW motion is highly coherent, approximating the qualitative behaviour expected in zero-temperature classical elastic models.

With this experimental picture we can understand why the most popular models of CDW transport have failed. Contrary to their assumption, there are two energy and two length scales in the problem, associated with local and collective pinning, respectively. These scales are widely separated, and yet both play crucial roles in determining the qualitative features of experiments.

What is the correct explanation of the I - V characteristic’s form? Some hints have been given [20,30,35], but a full theory of CDW dynamics in the presence of local and collective pinning and that also accounts for single particle screening is a subject for future work.

Acknowledgements

This work was sponsored by the U. S. National Science Foundation (NSF DMR 04-05500).

References

- [1] P. Monceau, N. P. Ong, A. M. Portis, A. Meerschaut, J. Rouxel, *Phys. Rev. Lett.* 37, 602 (1976).
- [2] R. M. Fleming, C. C. Grimes, *Phys. Rev. Lett.* 42, 1423 (1979).
- [3] N. P. Ong, J. W. Brill, J. C. Eckert, J. W. Savage, S. K. Khanna, R. B. Somoano, *Phys. Rev. Lett.* 42, 811 (1979).
- [4] H. Fukuyama, P. A. Lee, *Phys. Rev. B* 17, 535 (1978); P. A. Lee, T. M. Rice, *Phys. Rev. B* 19, 3970 (1979).
- [5] K. Maki, *Phys. Rev. Lett.* 39, 46 (1977).
- [6] J. Bardeen, *Phys. Rev. Lett.* 42, 1498 (1979); J. Bardeen, *ibid.* 45, 1978 (1980).
- [7] R. E. Thorne, J. H. Miller, Jr., W. G. Lyons, J. W. Lyding, J. R. Tucker, *Phys. Rev. Lett.* 55, 1006 (1985); R. E. Thorne, J. R. Tucker and J. Bardeen, *Phys. Rev. Lett.* 58, 828 (1987); R. E. Thorne, Ph.D. thesis, University of Illinois, Urbana (1987).
- [8] L. Sneddon, M. C. Cross, D. S. Fisher, *Phys. Rev. Lett.* 49, 292 (1982).
- [9] H. Matsukawa, H. Takayama, *J. Phys. Soc. Jpn.* 56, 1507 (1987); H. Matsukawa, *ibid.* 57, 3463 (1988).
- [10] J. McCarten, M. Maher, T. L. Adelman, R. E. Thorne, *Phys. Rev. Lett.* 63, 2841 (1989).
- [11] D. S. Fisher, *Phys. Rev. Lett.* 50, 1486 (1983); D. S. Fisher, *Phys. Rev. B* 31, 1396 (1985).
- [12] P. Monceau, M. Renard, J. Richard, M. C. Saint-Lager, Z. Z. Wang, in *Charge Density Waves in Solids*, edited by Gy. Hutiray, J. Solyom (Springer-Verlag, Berlin, 1985), p. 275; J. C. Gill, *ibid.* p. 377.
- [13] M. O. Robbins, J. P. Stokes, S. Bhattacharya, *Phys. Rev. Lett.* 55, 2822 (1985); S. Bhattacharya, M. J. Higgins, J. P. Stokes, *Phys. Rev. Lett.* 63, 1503 (1989).
- [14] M. P. Maher, T. L. Adelman, J. McCarten, D. A. DiCarlo, R. E. Thorne, *Phys. Rev. B* 43, 9968 (1991); S. G. Lemay, M. C. de Lind van Wijngaarden, T. L. Adelman, R. E. Thorne, *Phys. Rev. B* 57, 12781 (1998).
- [15] R. E. Thorne, J. S. Hubacek, W. G. Lyons, J. W. Lyding, J. R. Tucker, *Phys. Rev. B* 37, 10055 (1988).
- [16] O. Narayan, D. S. Fisher, *Phys. Rev. Lett.* 68, 3615 (1992).
- [17] A. A. Middleton, O. Biham, P. B. Littlewood, P. Sibani, *Phys. Rev. Lett.* 68, 1586 (1992); A. A. Middleton, Ph.D. Thesis, Princeton University, 1991.
- [18] C. R. Myers, J. P. Sethna, *Phys. Rev. B* 47, 11194 (1993); C. R. Myers, Ph.D. Thesis, Cornell University, 1992.
- [19] For example, see L. Balents, M. P. A. Fisher, *Phys. Rev. Lett.* 75, 4270 (1995); L. Balents, M. C. Marchetti, L. Radzihovsky, *Phys. Rev. B* 57, 7705 (1998); K. Saunders, J. M. Schwarz, M. C. Marchetti, A. A. Middleton, *Phys. Rev. B* 70, 024205 (2004).
- [20] S. Brazovskii and T. Nattermann, *Adv. Phys.* 53, 177 (2004).
- [21] J. C. Gill, *Synth. Met.* 43, 3917 (1991); J. McCarten, M. Maher, T. L. Adelman, D. A. DiCarlo, R. E. Thorne, *Phys. Rev. B* 43, 6800 (1991).
- [22] A. Zettl, G. Gruner, *Phys. Rev. B* 26, 2298 (1982).
- [23] R. M. Fleming, R. J. Cava, L. M. Schneemeyer, E. A. Rietman, R. G. Dunn, *Phys. Rev. B* 33, 5450 (1986).
- [24] G. Mihaly, T. Chen, T. W. Kim, G. Gruner, *Phys. Rev. B* 38, 3602 (1988).
- [25] M. E. Itkis and F. Ya. Nad, *Synth. Metals* 41-43, 4037 (1991).
- [26] P. B. Littlewood, *Solid State Commun.* 65, 1347 (1988); P. B. Littlewood, *Synth. Metals* 29, F531 (1989).
- [27] F. Levy, M. Sherwin, F. Abraham, K. Wiesenfeld, *Phys. Rev. Lett.* 68, 2968 (1992).
- [28] R. P. Hall, M. F. Hundley, A. Zettl, *Phys. Rev. B* 38, 13002 (1988).
- [29] T. L. Adelman, J. McCarten, M. P. Maher, D. A. DiCarlo, R. E. Thorne, *Phys. Rev. B* 47, 4033 (1993).

- [30] S. G. Lemay, R. E. Thorne, Y. Li, J. D. Brock, *Phys. Rev. Lett.* 83, 2793 (1999).
- [31] S. G. Lemay, K. O'Neill, C. Cicak, R. E. Thorne, *Phys. Rev. B* 63, 081102 (2001).
- [32] N. Ogawa, A. Shiraga, R. Kondo, S. Kagoshima, K. Miyano, *Phys. Rev. Lett.* 87, 256401 (2001);
M. Ogawa, K. Miyano, S. Brazovski, *Phys. Rev. B* 71, 075118 (2005).
- [33] S. V. Zaitsev-Zotov (private communication).
- [34] K. Cicak, K. O'Neill, S. G. Lemay, R. E. Thorne (in preparation.)
- [35] S. Brazovskii, A. Larkin, *Synth. Metals* 86, 2223 (1997).