This experiment uses multiple scintillators and a logic array to determine the decay rate of cosmic ray muons in vacuum. Data was collected over a period of 406.5 hours. The lifetime of the muon was measured to be $2.178 \pm 0.077 \mu s$, in good agreement with the current best value of $2.19703 \pm 0.00004 \mu s$ [1]. Corrections for a shorter $\mu^-$ lifetime in scintillator and an unequal number of $\mu^+$ and $\mu^-$ particles have been taken into account.

INTRODUCTION

Carl Anderson discovered the muon while studying cosmic radiation in 1937 [2]. Originally identified as the “mu meson,” it was later determined that, unlike mesons, the particle did not interact strongly with matter, so the name was shortened to “muon.” With elementary charge $-1$ and mass $105.6 \text{ MeV} / c^2$, about 207 times the electron mass, the muon is actually part of the lepton family of fermions. This family includes the electron, tau, and three types of neutrinos.

The study of muon decay is important for many reasons. Investigations into its decay provide a test for predictions of the Standard Model. First, muon decay led to the discovery of the muon neutrino, $\nu_\mu$, for which three researchers were awarded the 1988 Nobel Prize in Physics. In addition, muon decay demonstrates parity violation [3]. Muons are polarized anti-parallel to their flight direction, and they retain this polarization when stopping in the scintillator. This results in an anisotropic distribution of emitted decay electrons. Finally, the muon lifetime can be calculated using the Electroweak Unified Theory, which gives

$$\tau_\mu = \frac{192\pi^3 h^7}{G_F m_\mu c^8}$$

where $G_F$ is the fundamental Fermi coupling constant [3]. A Feynman diagram illustrating the muon decay through the weak interaction is given in Figure 1.

The muons used in this experiment originate from cosmic rays entering the Earth’s atmosphere. Primary cosmic rays are charged particles and bare nuclei of the elements that have very high energy. Cosmic rays have been observed with energies per particle in excess of $10^{20} \text{ eV}$ [4]. Approximately 90% of cosmic rays are protons, 9% Helium nuclei, and 1% other. These cosmic rays interact with particles in the upper atmosphere at an altitude above 15 km and create pions (among other things) [5]. Pions decay with a lifetime of $2.6033 \times 10^{-8} \text{ seconds}$ [6]. Negatively charged pions ($\pi^-$) and positively charged pions ($\pi^+$) decay into a muon ($\mu^-$) and antimuon ($\mu^+$), respectively, via the decay modes

$$\pi^+ \rightarrow \mu^+ + \nu_\mu,$$  \hspace{1cm} (2)

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu,$$  \hspace{1cm} (3)

where $\nu_\mu$ is a muon-neutrino and $\bar{\nu}_\mu$ is its antiparticle. Figure 2 illustrates this process of creating muons from cosmic rays.

Muons themselves are unstable, with a lifetime of about $2.2 \mu s$. By CPT invariance, the positive and negative muon must have the same mass and lifetime in vacuum, because one is the antiparticle of the other. The major decay mode for each muon is given by

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu,$$  \hspace{1cm} (4)

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.$$  \hspace{1cm} (5)

Due to the extremely short lifetime of pions, they do not travel more than a few hundred kilometers before decaying into muons. Therefore, muons are being created at altitudes around 15 km. Given that the muon lifetime is only $2.2 \mu s$, Newtonian kinematics would demand that the muons have a velocity greater than $20c$ to reach the surface of the Earth. If the muons were traveling at the speed of light, it would take them $50 \mu s$ to traverse the distance. This implies that relativistic effects come into play, namely in the form of length contraction and time dilation. In the laboratory frame, the muon...
FIG. 2: A primary cosmic ray proton interacts with atmospheric particles to create this shower of particles. In the process, pions are created which then decay into the muons that we detect in the laboratory.

The lifetime appears time dilated by a factor of \( \gamma \), where 
\[
\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.
\]
For a muon with velocity 0.999c, \( \gamma \approx 22 \), and the apparent muon lifetime is large enough to account for its detection at sea level. The effect can also be explained using the reference frame of the muon. In this frame, the muon can reach the surface because the distance it must travel is Lorentz contracted by a factor of \( \gamma \).

For this experiment, the situation is complicated by the fact that the muon lifetime is measured in scintillator, not vacuum. The scintillator is composed mostly of carbon and hydrogen molecules, and the carbon nuclei can capture the muons. Positive muons may interact with the molecules to form muonium \( (\mu^+e^-) \), an exotic atom similar to hydrogen, according to the process
\[
\mu^+ + X \rightarrow (\mu^+e^-) + X^+,
\]
where \( X \) is some available molecule. After muonium is formed, it decays through another allowed decay mode
\[
\mu^+ + e^- \rightarrow \nu_e + \bar{\nu}_\mu
\]
for positive muons. The branching ratio for (6) is 1 part in \( 10^{10} \) [7]. Therefore, muonium formation does not appreciably affect the decay rate of \( \mu^+ \).

However, there is a correction for the decay rate of \( \mu^- \) in scintillator. When negative muons stop in matter, they can form a "muonic atom" through the process
\[
\mu^- + X \rightarrow (\mu^-X^+) + e^-.
\]

Since the muon mass is much greater than that of the electron, the muon has a radius that is a factor of 207 smaller than the Bohr radius. This facilitates direct interaction between the protons of the carbon nuclei and \( \mu^- \), resulting in the decay
\[
\mu^- + p \rightarrow n + \nu_\mu.
\]

The capture of negative muons (8) results in the electron emission, mimicking the signal of a muon decay. Thus, negative muons have an apparently shorter lifetime than positive muons. Since the decay rates for these two particles are unequal, we must also consider the unequal number of positive and negative muons incident upon the scintillator.

**EXPERIMENTAL SETUP**

The layout for this experiment is shown in Figure 3. As the muons travel through the atmosphere, they lose energy. Those with kinetic energy less than 100 MeV will have low enough energy to stop in the scintillator. When they stop, the scintillator outputs an amount of light dependent on the amount of energy deposited by the incident particle. This light is measured using a photomultiplier viewing the scintillator. The PMT is controlled by the high voltage setting. The high voltage must be set to an optimal value for obtaining the best data. Applying a voltage too high results in excess noise, while...
a second decay pulse is detected within 25 µsec of the first stop- 
ing pulse, the 25 µsec delay. A timing diagram of the system is given in Figure 4.

Associated with each decay event is the time interval between the stopping and decay pulse. Each of these decay times are stored as a number of counts in 256 time bins, each of width 0.1 µsec. By creating a histogram of the number of decays for each bin we find that the number of decays falls exponentially for increasing time intervals. In addition to the exponential decay, there is a uniform background of counts. This background results from random coincidences due to the finite probability of two muons stopping in the scintillator in some time interval. Thus, these events mimic a decay event, even though no decay has occurred. This background is expected to be random and relatively small. We then extract the muon lifetime in vacuum from these measurements.

RESULTS AND DISCUSSION

Without the corrections mentioned in the Introduction section, the muon lifetime could be determined directly from the exponential decrease in the number of counts. However, due to the unequal lifetimes of the positive and negative muons, the best-fit curve to the data should be composed of two exponential decays in addition to a constant background. Taking these corrections into account, and using the fact that each bin has a finite size, we find that the number of counts in a given bin to be

\[ N_{\text{bin}}(t) = B - \Delta t \frac{d}{dt} N_0 \left( \frac{e^{-\lambda t}}{1 + r} + \frac{e^{-(\lambda + \Delta) t}}{1 + r} \right) \]  

(10)
to first order, where \( t = t_{\text{bin}} \), \( \lambda \) is the free decay rate of muons (equal to the inverse of the muon lifetime), \( B \) is the number of background counts per bin, \( \Delta t \) is the bin width, \( N_0 \) is number of muons at time \( t = 0 \), \( r \) is the ratio \( N_{\mu^+} : N_{\mu^-} \), and \( \Lambda \) is the capture rate of \( \mu^- \) in scintillator [8].

The experiment ran for 406.5 hours, resulting in 45,205 double counts and 13,326,245 single counts. Therefore, the high voltage setting of 625 Volts yielded a counting rate of 9,107 counts/sec. Assuming a random, low background rate, we expect the number of background counts to be given by

\[ B = \Gamma^2 T \Delta t, \]  

(11)

where \( \Gamma \) is the counting rate and \( T \) is the total run time. Without having a direct way to test the clock, we cannot confirm whether or not the bin width is exactly 0.1 µsec. Using (11), we are not required to make any assumptions about \( \Delta t \). This is preferred, because any deviation in \( \Delta t \) would lead to a different value of the muon lifetime. By solving (11) for \( \Delta t \), this expression can be substituted into (10). Then a fit of (10) will yield best values for parameters \( \lambda \), B, and \( N_0 \).

If the measured number of decays are Gaussian-distributed, it would be sufficient to use the least-squares fitting method. Thus, one could find the parameters corresponding to the best-fit curve by minimizing \( \chi^2 \) with respect to each parameter [9]. However, this was a counting experiment which involved measuring the number of decays in a given time bin. Therefore, the measured number of counts in each bin, \( N_i \), is a Poisson distribution with \( \sigma_i = \sqrt{N_i} \). The method of maximum-likelihood is required in order to obtain the best parameters for the fit.
TABLE I: Parameters Found Using Maximum-Likelihood

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Error$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^b$ (usec)</td>
<td>2.178</td>
<td>0.077</td>
</tr>
<tr>
<td>B (Counts)</td>
<td>11.90</td>
<td>0.43</td>
</tr>
<tr>
<td>$N_a$ (Counts)</td>
<td>37760</td>
<td>210</td>
</tr>
</tbody>
</table>

$^a$Error found using error-matrix technique.
$^b$$\lambda = \frac{1}{\tau}$.

For $m$ measurements of the number of counts $N_a$ in a given time bin $t_a$, use of Poisson statistics in the maximum-likelihood calculation determined parameters corresponding to the best-fit curve from solutions of the three simultaneous equations [10]

$$\sum_{a=1}^{m} \frac{\partial N_{bin}(t_a)}{\partial \alpha_i} = \sum_{a=1}^{m} \frac{N_a}{N_{bin}(t_a)} \frac{\partial N_{bin}(t_a)}{\partial \alpha_i}$$

(12)

where $\alpha_i$ is one of the three parameters $\lambda$, $B$, or $N_a$. Given that $r = 1.18\pm0.12$ and $\Lambda = 3.76\pm0.04 \times 10^4$ sec$^{-1}$ [1], Table I summarizes the results using Mathematica to solve (12). Most importantly, we measure the lifetime of the muon to be $\tau_{\mu} = 2.178 \pm 0.077$ usec, where the error is determined through the error-matrix technique [10].

Figures 5 and 6 show comparisons between the data and fit (10) with the parameters in Table I. Figure 5 shows how both the fit and the data drop exponentially to a constant background value. The data points represent the number of counts per bin. Figure 6 is a plot of logarithm of the background-subtracted fit with data. Without correction for the muon lifetime in scintillator, the slope of this line could be used to directly determine the muon lifetime. The random scattering of data points above and below the fit qualitatively demonstrates the goodness of the fit.

As a consistency check of the parameters in Table I, the number of background counts is estimated directly from the data. Using the data points after 15 usec, when the curve has levelled off, the weighted average of the background is determined to be $11.87 \pm 0.54$ counts. This is in agreement with $11.90 \pm 0.43$ counts, the value found using maximum-likelihood. However, it should be noted that there is some subjectivity in determining where the background counts begin. The method of maximum-likelihood does not have this subjectivity: all of the data is used at once, without imposing arbitrary bounds.

For a more quantitative indication of the accuracy of the fit in describing the data points, we find a value of reduced chi squared, $\chi^2$ [9]. When taking into account the number of degrees of freedom from the number of measurements and constraints, we find a value of $\chi^2 = 0.940$. Thus, if the measurements really do follow the assumed distribution, then the probability of obtaining a value of $\chi^2$ as large as 0.940 for our number of degrees of freedom is 0.71, indicating the observed and expected distributions are consistent.

Finally, it should be noted that the discussion thus far has not taken into account the fact that the values of $r$ and $\Lambda$ are not exact quantities; they have some error associated with them. We consider them contributions to systematic error. By using the largest value of $r$ and the smallest value of $\Lambda$, we find that $\tau_{\mu} = 2.173$ usec. Conversely, using the smallest value of $r$ and the largest value of $\Lambda$, we find that $\tau_{\mu} = 2.183$ usec. Therefore, we see that this systematic error is 0.005 usec, a factor of...
15 smaller than the random error. By no means was this the only source of systematic error. Another source of systematic error stems from assuming no non-random background counts. It is possible that non-background events can occur from electronic pickup and glitches. The experiment is designed to have a minimal amount of systematic error, but one can never remove systematic error completely.

CONCLUSION

The muon serves as an excellent specimen for nuclear decay experiments. These experiments are straightforward enough to be accessible at the graduate or undergraduate level. At the same time, they provide a realistic introduction to important concepts in experimental modern physics such as non-Gaussian distributions and nonlinear fitting, and useful equipment including scintillator detectors and photomultiplier tubes.

This is a well-designed experiment, capable of very accurate and precise measurements of the muon lifetime. Using this apparatus, the lifetime of the muon is measured to be $2.178 \pm 0.077 \mu sec$, with estimated additional uncertainty 0.005 $\mu sec$ from systematic errors. This value is in good agreement with the current best experimental value of $2.19703 \pm 0.00004 \mu sec$ [1].

One improvement to this experiment would involve increasing the high voltage setting. This would yield a much larger counting rate than was achieved, resulting in a larger data sample. Random, statistical error is the largest contributor to the uncertainty on the lifetime measurement, and this could be reduced by getting more counts in each bin.

In addition, there are ways in which this experiment could be expanded. From a theoretical perspective, one could improve upon some of the assumptions and approximations made. For example, the next order correction to the fit (10) could be calculated to determine its effect. One could also see if a more exact form of the background (11) yields a discernable modification to the muon lifetime. From an experimental perspective, it would be more thorough to directly test the clock controlling the counter.

Acknowledgments

I would like to thank Professor Hand for our discussions about this experiment and possible improvements upon it. In addition, I would like to thank Robin Smith for her helpful suggestions concerning this report. As always, special thanks to Tien Le for her continual support.