Wave Interference

or

Measuring Path Differences
Wave model of Light

• Treat light as a classical electromagnetic wave
• Waves can add constructively or destructively
Interference and Diffraction

• All interference and diffraction patterns
  – Locate the two waves that are interfering
  – Determine path difference $d$
  – Determine phase change $\varphi$
    • Constructive interference if $\varphi$ is even multiple of $\pi$
    • Destructive interference if $\varphi$ is odd multiple of $\pi$

$$\frac{\varphi}{2\pi} = \frac{d}{\lambda}$$
Interfaces

• Wave may pass between media with different indexes of refraction $n_{\text{in}} \neq n_{\text{out}}$

• What changes?
  – Wave speed? Frequency? Wavelength?

• Reflection at interfaces
  – For wave passing into a region with a larger $n$, the reflection may be inverted
  – Need to add $\pi$ to the phase difference: $\phi \rightarrow \phi + \pi$
Thin Film Interference

• Beam splits at first interface
• Impedance mismatch at front interface (phase shift)
• There may also be a second mismatch at the back interface
• Path difference is twice film thickness:
  \[-d = 2w\]
  \[-(\phi+\pi)/2\pi = 2w/\lambda\]
82. Consider two horizontal glass plates with a thin film of air between them. For what values of the thickness of the film of air will the film, as seen by reflected light, appear bright if it is illuminated normally from above by blue light of wavelength 488 nanometers?

(A) 0, 122 nm, 244 nm
(B) 0, 122 nm, 366 nm
(C) 0, 244 nm, 488 nm
(D) 122 nm, 244 nm, 366 nm
(E) 122 nm, 366 nm, 610 nm
Bragg Diffraction

- Incident light enters crystal and rebounds at an angle, forming distinct diffraction points.
- Treat lattice planes of crystal as diffraction slits (or as thin film, at an angle).

\[ n\lambda = 2d \sin \theta \]

*Bragg’s Law*
Bragg Diffraction

- Incident light enters crystal and rebounds at an angle, forming distinct diffraction points
- Treat lattice planes of crystal as diffraction slits (or as thin film, at an angle)

When a narrow beam of monoenergetic electrons impinges on the surface of a single metal crystal at an angle of 30 degrees with the plane of the surface, first-order reflection is observed. If the spacing of the reflecting crystal planes is known from x-ray measurements to be 3 Ångstroms, the speed of the electrons is most nearly

\( n\lambda = 2d \sin \theta \)

Bragg’s Law

(A) \( 1.4 \times 10^{-4} \) m/s
(B) \( 2.4 \) m/s
(C) \( 5.0 \times 10^{3} \) m/s
(D) \( 2.4 \times 10^{6} \) m/s
(E) \( 4.5 \times 10^{9} \) m/s
Double Slit Diffraction

- **Path difference:**
  - Look at difference between light passing through two slits
  - Condition on maxima becomes \( d \sin \theta = n\lambda \)

\[ \tan \theta = \frac{y}{D} \]
\[ \tan \theta \sim \sin \theta \sim \theta \sim \frac{y}{D} \]

For distant screen assumption

\[ d \sin \theta = m\lambda \]
\[ y = \frac{m\lambda D}{d} \]

Assumption of infinite source distance gives plane wave at slit so that all amplitude elements are in phase.

For \( D \gg a \) this approaches a right angle and \( \theta' = \theta \)

\( a = \text{slit width} \)
Double Slit Diffraction

• Path difference:
  – Look at difference between light passing through two slits
  – Condition on maxima becomes \( d \sin \theta = n \lambda \)

70. Light from a laser falls on a pair of very narrow slits separated by 0.5 micrometer, and bright fringes separated by 1.0 millimeter are observed on a distant screen. If the frequency of the laser light is doubled, what will be the separation of the bright fringes?

(A) 0.25 mm
(B) 0.5 mm
(C) 1.0 mm
(D) 2.0 mm
(E) 2.5 mm

http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
Single Slit Diffraction

- Derivation – divide slit into N very narrow slits and taking N to infinity
- Diffraction pattern goes as \( \sin(\theta)/\theta \) with large central peak
  - Condition on **minima** becomes: \( a \sin \theta = m\lambda \)
  - Similar to double slit condition for maxima (easy to get confused)

57. Consider a single-slit diffraction pattern for a slit of width \( d \). It is observed that for light of wavelength 400 nanometers, the angle between the first minimum and the central maximum is \( 4 \times 10^{-3} \) radians. The value of \( d \) is

(A) \( 1 \times 10^{-5} \) m  
(B) \( 5 \times 10^{-5} \) m  
(C) \( 1 \times 10^{-4} \) m  
(D) \( 2 \times 10^{-4} \) m  
(E) \( 1 \times 10^{-3} \) m
Diffraction as Fourier Transform

• Diffraction patterns are Fourier transforms of the object
  – $\text{FT}[\text{two dirac deltas}] = \text{sine wave}$
  – $\text{FT}[\text{step function}] = \text{sinc function}$

• In practice, double slit pattern with finite widths is the convolution of sinc and sine
Example: Diffraction Patterns

- When does a minimum from the single slit cancel out a maximum from the double slit?
- Single-slit condition for minimum is same as double-slit condition for maximum

\[
\frac{\sin(\theta)}{\lambda} = \frac{m}{w} = \frac{n}{d}
\]

- Which answers are plausible?

20. In a double-slit interference experiment, \( d \) is the distance between the centers of the slits and \( w \) is the width of each slit, as shown in the figure above. For incident plane waves, an interference maximum on a distant screen will be "missing" when

(A) \( d = \sqrt{2w} \)
(B) \( d = \sqrt{3w} \)
(C) \( 2d = w \)
(D) \( 2d = 3w \)
(E) \( 3d = 2w \)
100. A Michelson interferometer is configured as a wavemeter, as shown in the figure above, so that a ratio of fringe counts may be used to compare the wavelengths of two lasers with high precision. When the mirror in the right arm of the interferometer is translated through a distance \( d \), 100,000 interference fringes pass across the detector for green light and 85,865 fringes pass across the detector for red (\( \lambda = 632.82 \) nanometers) light. The wavelength of the green laser light is

(A) 500.33 nm
(B) 543.37 nm
(C) 590.19 nm
(D) 736.99 nm
(E) 858.65 nm
Interferometry

- Each fringe represents constructive interference
- Occurs when change in path length equals integer number of wavelengths
- In this case: \(2d = m\lambda\) for each color of light

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Difficult Problems:
How to Make them Easy
95. A beam of $10^{12}$ protons per second is incident on a target containing $10^{20}$ nuclei per square centimeter. At an angle of 10 degrees, there are $10^2$ protons per second elastically scattered into a detector that subtends a solid angle of $10^{-4}$ steradians. What is the differential elastic scattering cross section, in units of square centimeters per steradian?

(A) $10^{-24}$  
(B) $10^{-25}$  
(C) $10^{-26}$  
(D) $10^{-27}$  
(E) $10^{-28}$
Nuclear/Particle/Experimental Physics

- Need units of answer to be cm²/steradian
- \((10^{20} \text{ cm}^2)^{-1} / 10^{-4} \text{ steradian} = 10^{-16} \text{ cm}^2/\text{steradian}\)
- Other two numbers given need to combine to be unitless
- \(\Rightarrow (10^2 \text{ proton/sec})/(10^{12} \text{ proton/sec}) = 10^{-10}\)
- Answer must be:
  \[10^{-16} \text{ cm}^2/\text{steradian} \times 10^{-10} = 10^{-26} \text{ cm}^2/\text{steradian}\]

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96. Which of the following reasons explains why a photon cannot decay to an electron and a positron \((\gamma \rightarrow e^+ + e^-)\) in free space?

(A) Linear momentum and energy are not both conserved.
(B) Linear momentum and angular momentum are not both conserved.
(C) Angular momentum and parity are not both conserved.
(D) Parity and strangeness are not both conserved.
(E) Charge and lepton number are not both conserved.
Particle Physics/Special Relativity

- Visualize this decay in the center of mass frame of the two electrons.
- Since the incident photon carries momentum, clearly momentum cannot be conserved.
- Can also work out the mathematics to show that linear momentum and energy cannot be simultaneously conserved here.

- (B) & (C): What angular momentum?
- (D): Strangeness? There are no quarks here! This is not a weak interaction.
- (E) Both charge and lepton number are conserved.

- Similarly, can the photoelectric effect occurring for a free electron?

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16. The mean free path for the molecules of a gas is approximately given by $\frac{1}{\eta \sigma}$, where $\eta$ is the number density and $\sigma$ is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) $10^{-4}$ m 
(B) $10^{-7}$ m 
(C) $10^{-10}$ m 
(D) $10^{-13}$ m 
(E) $10^{-16}$ m
Statistical Mechanics
Mean Free Path

• Quick way: know your length scales
Statistical Mechanics
Mean Free Path

• Quick way: know your length scales
  – $10^{-9} - 10^{-10}$ m is the size of an atom
  – $10^{-3}$ m is a millimeter, $10^{-5}$ m is the width of a human hair
Statistical Mechanics
Mean Free Path

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• Hard way: approximate $\sigma$, $\eta$ using ideal gas law
Quick way: know your length scales
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Hard way: approximate $\sigma$, $\eta$ using ideal gas law

$[\eta] = \text{atoms/volume}$

$$\eta = \frac{n}{V} = \frac{P}{RT} = \frac{1 \text{ atm}}{8 \cdot 10^{-5} \frac{m^3 \cdot \text{atm}}{\text{Mol} \cdot \text{K}}} \cdot 300 \text{K} \approx 1.2 \cdot 10^{25} \frac{\text{atom}}{m^3}$$

$$\sigma \approx (10^{-9})^2 m^2 \quad \frac{1}{\eta \sigma} \approx 10^{-25} \cdot 10^{18} m = 10^{-7} m$$
Mechanics
Curvature of Mars

22. The curvature of Mars is such that its surface drops a vertical distance of 2.0 meters for every 3600 meters tangent to the surface. In addition, the gravitational acceleration near its surface is 0.4 times that near the surface of Earth. What is the speed a golf ball would need to orbit Mars near the surface, ignoring the effects of air resistance?

(A) 0.9 km/s
(B) 1.8 km/s
(C) 3.6 km/s
(D) 4.5 km/s
(E) 5.4 km/s
Mechanics
Curvature of Mars

- Imagine a circular orbit using this (very not to scale) image:
- It takes a certain amount of time to fall 2 m under gravity = .4g
- In that same amount of time, the golf ball goes 3600 m
- The rest is just kinematics:

![](image)

v = ?

3600 m

2 m

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- It takes a certain amount of time to fall 2 m under gravity=.4g
- In that same amount of time, the golf ball goes 3600 m
- The rest is just kinematics:

\[ \Delta y = \frac{1}{2} \cdot 0.4g \left( \frac{\Delta x}{v} \right)^2, \quad \Delta x = v \Delta t \]

\[ \Delta y = \frac{1}{2} \cdot 0.4g \left( \frac{\Delta x}{v} \right)^2, \quad \Delta y = \frac{1}{2} \cdot 0.4g \frac{\Delta x^2}{\Delta y} \]

\[ v^2 = \frac{1}{2} \cdot 0.4 \frac{(3600)^2}{2}, \quad v = 3600 \text{ m/s} \]

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(E) 5.4 km/s
Waves – Standing Waves

85. Small-amplitude standing waves of wavelength $\lambda$ occur on a string with tension $T$, mass per unit length $\mu$, and length $L$. One end of the string is fixed and the other end is attached to a ring of mass $M$ that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength? (You might want to consider the limiting cases $M \to 0$ and $M \to \infty$.)

(A) $\mu/M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B) $\mu/M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

(C) $\mu/M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

(D) $\lambda = 2L/n, \ n = 1, 2, 3, \ldots$

(E) $\lambda = 2L/(n + \frac{1}{2}), \ n = 1, 2, 3, \ldots$
## Waves – Standing Waves

- Treat as standing waves
- Take limits! How does the condition at the boundary change when…
  - $M \rightarrow \infty$: closed boundary
  - $M \rightarrow 0$: open boundary

### $M \rightarrow \infty$
- Right hand side must go to zero at allowed frequencies

### $M \rightarrow 0$
- Right hand side must go to $\infty$ at allowed frequencies

<table>
<thead>
<tr>
<th>Harmonic</th>
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<th>Frequency $f$</th>
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<tr>
<td>1$^{st}$</td>
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<td>$f_1$</td>
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<tr>
<td>2$^{nd}$</td>
<td>$L$</td>
<td>$2f_1$</td>
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<tr>
<td>3$^{rd}$</td>
<td>$2L/3$</td>
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Odd and Even Harmonics

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Odd Harmonics
Electromagnetism

- In case you were worried about wave guides

34. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

(A) $E_n = 0, B_n = 0$

(B) $E_n = 0, B_t = 0$

(C) $E_t = 0, B_t = 0$

(D) $E_t = 0, B_n = 0$

(E) None of the above
Recall Maxwell’s equations:

1. \[ \iint \vec{E} \cdot \hat{n} \, dS = Q_f \]

2. \[ \iint \vec{B} \cdot \hat{n} \, dS = 0 \]

3. \[ \oint \vec{E} \cdot d\ell = -\frac{d}{dt} \iint \vec{B} \cdot \hat{n} \, dA \]

4. \[ \oint \vec{B} \cdot d\ell = I_f + \frac{l}{c^2} \iint \frac{\partial}{\partial t} \, \vec{E} \cdot \hat{n} \, dA \]

Draw a small Amperian loop containing the interface:

- Using Faraday’s law (#3), as the loop becomes very small the flux vanishes, we find that the tangential electric field must be continuous at the boundary:

\[ \vec{E}_{out} \cdot \vec{l} - \vec{E}_{in} \cdot \vec{l} = 0 \Rightarrow E_{out}^\parallel - E_{in}^\parallel = 0 \]

Draw a small Gaussian pillbox containing the interface:

- Applying (#2), we find that the perpendicular magnetic field must be continuous across the boundary:

\[ \vec{B}_{out} \cdot \hat{n} - \vec{B}_{in} \cdot \hat{n} = 0 \Rightarrow B_{out}^\perp - B_{in}^\perp = 0 \]
• Returning to the problem, we can remember that electromagnetic waves are shielded by conductors, so we set $E$ and $B$ to 0 on the inside.

• We can now obtain a rule for the boundary condition at the cavity wall

• In general, even in the presence of sources, we can use all four of Maxwell’s equations to obtain four possible boundary conditions:

1. $\epsilon_{\text{out}} E_{\text{out}}^{\perp} - \epsilon_{\text{in}} E_{\text{in}}^{\perp} = \sigma_f$

2. $B_{\text{out}}^{\perp} - B_{\text{in}}^{\perp} = 0$

3. $E_{\text{out}}^{\parallel} - E_{\text{in}}^{\parallel} = 0$

4. $\frac{1}{\mu_{\text{out}}} B_{\text{out}}^{\parallel} - \frac{1}{\mu_{\text{in}}} B_{\text{in}}^{\parallel} = \overrightarrow{K}_f \cdot \hat{n}$

34. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

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(B) $E_n = 0, B_t = 0$

(C) $E_t = 0, B_t = 0$

(D) $E_t = 0, B_n = 0$

(E) None of the above

Griffiths, *Introduction to Electrodynamics*, 7.3.6: “Boundary Conditions”
87. Two small pith balls, each carrying a charge \( q \), are attached to the ends of a light rod of length \( d \), which is suspended from the ceiling by a thin torsion-free fiber, as shown in the figure above. There is a uniform magnetic field \( B \), pointing straight down, in the cylindrical region of radius \( R \) around the fiber. The system is initially at rest. If the magnetic field is turned off, which of the following describes what happens to the system?

(A) It rotates with angular momentum \( qBR^2 \).

(B) It rotates with angular momentum \( \frac{1}{4} qBd^2 \).

(C) It rotates with angular momentum \( \frac{1}{2} qBRd \).

(D) It does not rotate because to do so would violate conservation of angular momentum.

(E) It does not move because magnetic forces do no work.
Electromagnetism

• Hard way – Maxwell’s equations

1. $\oint \hat{E} \cdot \hat{n} \, dS = Q_f$

2. $\oint \hat{B} \cdot \hat{n} \, dS = 0$

3. $\int \hat{E} \cdot d\ell = -\frac{d}{dt} \oint \hat{B} \cdot \hat{n} \, dA$

4. $\oint \hat{B} \cdot d\ell = I_f + \frac{1}{c^2} \int \frac{\partial}{\partial t} \hat{E} \cdot \hat{n} \, dA$
Electromagnetism

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4. \( \oint \mathbf{B} \cdot d\ell = I_f + \frac{l}{c^2} \iint \frac{\partial}{\partial t} \mathbf{E} \cdot \hat{n} \, dA \)

• Which of these helps us here?
  
  – Faraday’s law – changing magnetic fields causes EMF
  
  – The magnetic fields are not doing work: it is the field induced by the changing magnetic field that does work on the charges
Electromagnetism

- Using Faraday’s law
  - Integrate both sides
  \[ E\pi d = -\frac{d}{dt}(B\pi R^2) \]

- Change in angular momentum comes from torque on rod
  \[ \Delta L = \int \tau dt = \int 2qE(d/2)dt = \int qEddt \]

- Integrate final expression for electric field to find total change in L
  \[ \Delta L = \int -\frac{d}{dt}(qBR^2)dt = qBR^2 \]
Electromagnetism

• The quick way: take limits
  - How should $\Delta L$ change as $R \rightarrow 0$?
  - How should $\Delta L$ change as $d \rightarrow \infty$?

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Electromagnetism

- Dielectrics change electric fields

54. A dielectric of dielectric constant $K$ is placed in contact with a conductor having surface charge density $\sigma$, as shown above. What is the polarization (bound) charge density $\sigma_p$ on the surface of the dielectric at the interface between the two materials?

(A) $\sigma \frac{K}{1-K}$  (B) $\sigma \frac{K}{1+K}$  (C) $\sigma K$

(D) $\sigma \frac{1+K}{K}$  (E) $\sigma \frac{1-K}{K}$
Electromagnetism

- Definition of dielectric constant $K$
  - Permittivity increases in matter
    \[ \varepsilon = K\varepsilon_0 > \varepsilon_0 \]

- What is the sign of the bound charge induced in the dielectric?
- Try taking limits:
  - What happens if $K=1$?
  - No dielectric, just vacuum
  - No polarization, no bound charge!

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(D) $\sigma \frac{1+K}{K}$  (E) $\sigma \frac{1-K}{K}$
80. A tube of water is traveling at $1/2 \, c$ relative to the lab frame when a beam of light traveling in the same direction as the tube enters it. What is the speed of light in the water relative to the lab frame? (The index of refraction of water is $4/3$.)

(A) $1/2 \, c$
(B) $2/3 \, c$
(C) $5/6 \, c$
(D) $10/11 \, c$
(E) $c$
Special Relativity

• In the moving frame of the tube, the light is only moving \( \frac{c}{n} = \frac{3}{4}c < c \), so let’s just treat this like an ordinary velocity addition problem (\( c = 1 \)):

\[
v' = \frac{u+v}{1+uv}
\]

• \( u \) is the velocity of the tube with respect to the lab frame
• \( v \) is the velocity of light within the tube

• Since \( u \) and \( v \) have the same sign in the lab frame, they have the same sign when using this formula (they would have opposite sign if the tube were moving in opposite direction of light

80. A tube of water is traveling at \( \frac{1}{2} c \) relative to the lab frame when a beam of light traveling in the same direction as the tube enters it. What is the speed of light in the water relative to the lab frame? (The index of refraction of water is \( \frac{4}{3} \).)

(A) \( \frac{1}{2} c \)
(B) \( \frac{2}{3} c \)
(C) \( \frac{5}{6} c \)
(D) \( \frac{10}{11} c \)
(E) \( c \)
49. The infinite $xy$-plane is a nonconducting surface, with surface charge density $\sigma$, as measured by an observer at rest on the surface. A second observer moves with velocity $v\hat{x}$ relative to the surface, at height $h$ above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\frac{\sigma}{2\epsilon_0} \hat{z}$

(B) $\frac{\sigma}{2\epsilon_0} \sqrt{1 - \frac{v^2}{c^2}} \hat{z}$

(C) $\frac{\sigma}{2\epsilon_0 \sqrt{1 - \frac{v^2}{c^2}}} \hat{z}$

(D) $\frac{\sigma}{2\epsilon_0} \left( \sqrt{1 - \frac{v^2}{c^2}} \hat{z} + \frac{v}{c} \hat{x} \right)$

(E) $\frac{\sigma}{2\epsilon_0} \left( \sqrt{1 - \frac{v^2}{c^2}} \hat{z} - \frac{v}{c} \hat{y} \right)$
Special Relativity/Electrodynamics

• Imagine drawing the electric field lines in the lab frame as being spaced evenly apart
• Now imagine boosting to the moving observer’s frame
• Now the charged plane appears to be moving at \(-vx\)
• How do the distances between the evenly spaced electric field lines change in the moving frame?
  – **Lengths contract** - field lines are *denser* and the electric field must appear *stronger* to the moving observer
  – Observer sees more charge in a given amount of time if the plane is moving than if the plane is stationary
Special Relativity/Electrodynamics

• Other considerations:
  – The answer must reflect a stronger field
  – Also, no symmetry breaking: the field is not bent in the y or x directions

• Can also remember Lorentz-transformed fields

\[
\begin{align*}
  E'_x &= E_x \\
  E'_y &= \gamma(E_y - \beta B_z) \\
  E'_z &= \gamma(E_z + \beta B_y) \\
  B'_x &= B_x \\
  B'_y &= \gamma(B_y + \beta E_z) \\
  B'_z &= \gamma(B_z - \beta E_y)
\end{align*}
\]