ACCOUNTING FOR FLUCTUATIONS IN STOCHASTIC SIRS MODEL ON NETWORKS

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INTRODUCTION

Here we present our analysis of the stochastic Susceptible-Infected-Recovered-Susceptible (SIRS) model of infectious disease dynamics on heterogeneous networks. We perform a moment closure analysis to obtain approximate analytical predictions for the magnitude of fluctuations in the endemic state. We use the heterogeneous mean field (HMF) to approximate dynamics on a network with degree heterogeneity. This can be used to show how stochastic fluctuations and spontaneous extinction depend on the size and heterogeneity of the full network. We present results from our simulations to demonstrate the accuracy and utility of our predictions.

SIRS MODEL OVERVIEW

**Waning Immunity**

Infection spreads through contact; infected hosts develop immunity; recovered individuals lose immunity over time [1]

\[
\text{Infection by neighbor} \quad \text{Recovery}
\]

\[\begin{array}{c c c}
S & \beta I & R \\
\text{Waning Immunity } \rho & \gamma & \\
\end{array}\]

**Similarity to other models**

- SIRS \(\rightarrow\) SIR as \(\rho \rightarrow 0\) — Disease dies out after initial outbreak
- SIRS \(\rightarrow\) SIS as \(\rho \rightarrow \infty\) — Persistent endemic disease
- When does endemic disease persist in the SIRS model?

On Networks

- Each node represents a single individual.
- Each node is in one of three states (S, I, R)
- Network defines contacts between individuals
- Network structure accounts for contact heterogeneity [2]

HETEROGENEOUS MEAN FIELD THEORY

- Assume no degree correlations in network
- Partition graph into degree classes, classes couple together
- Account for degree heterogeneity in the mean field

\[
\frac{dS_k}{dt} = -\beta S_k I_k + \rho (1 - S_k - I_k), \quad \frac{dI_k}{dt} = \beta S_k I_k - \gamma I_k
\]

\[
\Theta_k = \sum_{k'} P(k'|k) I_{k'}, \quad \text{Force of infection between degree classes } k \& k'
\]

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MOMENT CLOSURE

We obtain fluctuation magnitudes by solving for the quasi-stationary distribution (QSD) of the number of Infecteds in the population. We use Gaussian moment-closure to approximately solve for the QSD.

1. Derive Kolmogorov forward equation (KFE) for the model.
2. Condition on no extinction (> 0 infected individuals, not reaching the absorbing state) to obtain KFE for QSD [3].
3. Obtain PDE for the Probability Generating Function for the QSD.
4. Assume rate of extinction is small, ignore nonlinear sink term [4].
5. Obtain PDE for the Cumulant Generating Function \(K\).
6. Assume Gaussian form of QSD, quadratic \(K\):

\[K(\theta, \phi, \tau) = \mu_0 \theta + \mu_1 \phi + \mu_2 \phi \theta + \frac{1}{2} \sigma_0^2 \theta^2 + \frac{1}{2} \sigma_1^2 \phi^2 + \ldots\]

7. Collect terms in series expansion to derive coupled ODEs for all first and second moments of the QSD (means \(\mu_0\) and variances \(\sigma^2\)).
8. Numerically integrate ODEs for \(\mu_0\) and \(\sigma^2\) as functions of time.
9. Can look for stationary behavior of QSD, \(\mu_0\), and \(\sigma^2\).
10. Knowing the variances \(\sigma^2\) characterizes the magnitude of stochastic fluctuations. With this technique, we can account for both finite population size effects and node degree heterogeneity.

RESULTS

The network can support endemic disease (\(\mu_p > 0\)) when \(R_0 \equiv \beta(k)/\gamma\) and \(\rho/\gamma\) are sufficiently large. We plot the signal to noise ratio (SNR) of the number of infected individuals \(\mu_p/\sigma_p\) to show persistence of endemic disease. Network properties also affect endemic disease:

**Population Size** Predicted and simulated SNR \(\mu_p/\sigma_p\) for \(G(n, p)\)-type graphs of two different sizes:

**Degree Heterogeneity** Predicted and simulated SNR \(\mu_p/\sigma_p\) for a graph with a skewed degree distribution \((k)^2/k^3 = .44)\:

**FUTURE WORK**

- Measure how extinction rates are affected by both finite size and degree heterogeneity in graphs with simulations.
- Examine role of low-degree nodes: moment closure predicts they decrease fluctuations in graphs with skewed degree distributions.
- Find a perturbative correction to the initial approximation that accommodates cases where rate of extinction is not small.
- Are two degree classes sufficient to capture network heterogeneity?