Quiz 4: Currents as Sources of Magnetic Field

A long, straight wire carries a current $I$ out of the page, as depicted below. This current produces a magnetic field in the vicinity of the wire. In the plane of the page we draw an imaginary closed path composed of two straight segments ($a$ and $c$) and two semi-circles ($b$ and $d$) of radii $R_2$ and $R_1$. The orientation of this path is given by the arrow ($\rightarrow$) below $a$.

1. What is the magnitude of the magnetic field along segments $b$ and $d$?

2. Write down the expression for the magnetic field along segment $a$ in unit vector notation as a function of $x$ (the wire is located at $x = 0$, so $R_1 < x < R_2$).
3. Let $d\mathbf{s}$ be an infinitesimal line element along the path. For each segment $(a, b, c, \text{ and } d)$, determine whether the dot product $\mathbf{B} \cdot d\mathbf{s}$ is positive (+), negative (−), or zero (0).

4. For each segment $(a, b, c, \text{ and } d)$, compute the line integral

$$\int \mathbf{B} \cdot d\mathbf{s}$$

along that segment. Express your answer in terms of $I, R_1, R_2$, and fundamental constants.

5. Without using Ampère’s law, compute the closed line integral

$$\oint \mathbf{B} \cdot d\mathbf{s}$$

around the entire path.

6. What does Ampère’s law predict for the value of the previous integral? Does this system obey Ampère’s law?