

# **Resonant Origins for Pluto's High Inclination**

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## **Abstract**

The origin of Pluto's highly eccentric orbit has been attributed to capture in a 3:2 mean motion resonance with a migrating Neptune [1], but its large inclination has eluded explanation. Here, we find that the inclination resonance of the same type is incapable of capturing Pluto prior to the eccentricity type. We also studied a potential secular resonance in the nodal precession frequencies of Neptune and Pluto and found that such a resonance is able to capture despite the mean motion resonance and can indeed raise inclinations over long times. A similar resonance may also be present early in the Solar System's formation, raising the inclination significantly prior to migration. Future work will constrain the conditions needed for either of these resonances to produce the orbital characteristics observed today.

## I. INTRODUCTION

The highly inclined and eccentric orbit of Pluto, protected from close encounters by resonances with Neptune, is one of the most surprising features of the Solar System. A successful model explaining Pluto's eccentricity was put forward by Malhotra and postulates that Neptune was able to migrate away from the Sun by a preferential inward scattering of planetesimals early in the Solar System's history [1]. As Neptune's orbit approached Pluto's, the latter was caught in a 3:2 mean motion resonance. Once trapped, Pluto migrated outward with Neptune while increasing its eccentricity to the value that it holds today.

This model provides a reasonable explanation for both Pluto's anomalous eccentricity ( $e_P = 0.24$ ) and the presence of the observed 3:2 resonance without requiring fine tuning of the Solar System's initial conditions. All planets are assumed to begin in relatively circular and un-inclined orbits, which is consistent with models of Solar System formation, and no catastrophic encounters are required in order to produce changes in orbital elements. However, this model only addresses Pluto's eccentricity and leaves open the question of the origin of its high inclination ( $i_P = 17^\circ$ ). One possible suspect is the nearby 6:4 inclination-type mean motion resonance, but, being a second-order resonance, it is far weaker than the aforementioned first-order eccentricity-type resonance.

Another possibility is a resonance in the precession frequencies of the longitudes of the ascending nodes of Neptune and Pluto. A potential 2:1 resonance of this type was identified in Applegate et al.'s spectral analysis of a 200 Myr numerical simulation of the Solar System [2]. As this resonance involves the nodes of both planets, it has the ability to affect their inclinations. A careful study is needed, then, to explore this and related resonances before, during, and after the migration of Neptune and to determine the magnitude of their effects and the conditions required for them to be significant.

## II. BACKGROUND

In accordance with models by Fernández and Ip [3], we postulate that the outer planets formed in relatively flat, circular orbits, most of which were nearer to the Sun than their current positions. The space around the planets was full of planetesimals which could be scattered by the planets either towards or away from the Sun. On average, planetesimals scattered by bodies other than

TABLE I: Early migrations of the outer Solar System.

Planet	Jupiter	Saturn	Uranus	Neptune
$a_0$ [AU]	5.4	8.5	15	22
$a$ [AU]	5.2	9.5	20	30
$\Delta a$ [AU]	-0.2	+1.0	+5	+8

Jupiter would either be directed inward towards Jupiter or outwards with insufficient force to escape. Jupiter, on the other hand, was capable of scattering planetesimals outward with enough force to prevent their return. As a result, Saturn, Uranus, and Neptune preferentially scattered inward while Jupiter preferentially scattered outward. The momentum-conserving recoils from this scattering caused Saturn, Uranus, and Neptune to migrate significant distances outward while Jupiter migrated a small amount inward. When simulating the migrations of the Solar System, we aimed to reproduce migration parameters similar to those given in Table I.

Mean motion resonances, such as the one responsible for Pluto’s eccentricity, are best expressed in terms of the orbital elements of the two bodies. These elements contain the same information as the instantaneous position and velocity vectors for an orbiting body, but they do so in terms of parameters for elliptical solutions to the 2-body problem. Two of the elements, the semi-major axis  $a$  and the eccentricity  $e$ , describe the size and shape of the ellipse, while the inclination  $i$  expresses its orientation with respect to a reference plane. The orientation of the ellipse in absolute space is represented by the longitude of the ascending node  $\Omega$  (the angle with respect to the reference direction at which the orbit intersects the reference plane in the upwards direction) and the longitude of perihelion  $\varpi$  (the angle formed with the reference direction by the semi-major axis passing through perihelion). A related quantity is the argument of perihelion, given by  $\omega = \varpi - \Omega$ . Finally, the position of the body within this orbit is given by the mean longitude  $\lambda$ .

A resonance occurs when the periods associated with several of these elements are in small integer ratios. Various symmetries place limits on which resonant arguments are valid [4]. The arbitrariness with which the reference direction is chosen requires that a resonant argument be invariant under the symmetry  $(\Omega, \varpi, \lambda) \rightarrow (\Omega + \theta, \varpi + \theta, \lambda + \theta)$ , where  $\theta$  is any angle. The result of this constraint is that the coefficients of all longitude angles in a resonant argument must sum to zero. Additional reflective symmetries also imply that the sum of the coefficients of all  $\Omega$  terms must be even. Examples of resonant arguments applicable to the Neptune+Pluto system are

$3\lambda_P - 2\lambda_N - \varpi_P$  (a first-order resonance involving only one longitude of perihelion) and  $6\lambda_P - 4\lambda_N - 2\Omega_P$  (a second-order resonance involving two longitudes of ascending nodes).

The application of linear secular theory [5] to an orbiting system is facilitated by working with the variables  $h$ ,  $k$ ,  $p$ , and  $q$ , defined as

$$\begin{aligned} h &= e \sin(\varpi) & k &= e \cos(\varpi) \\ p &= \sin(i/2) \sin(\Omega) & q &= \sin(i/2) \cos(\Omega) \end{aligned} \quad (1)$$

In the limit of small eccentricities and small inclinations, orbital theory becomes linear in these variables, and their spectra are characterized by distinct eigenfrequencies. A four-planet system, for example, oscillates with 4 distinct radial frequencies and 4 distinct vertical frequencies, known collectively as normal modes. The normal modes of  $h$  and  $k$  are denoted  $f_j$ , while the normal modes of  $p$  and  $q$  are denoted  $g_j$  (this follows the convention of Applegate et al., which is opposite that of Murray & Dermott). Conservation of angular momentum dictates that one of the normal modes of  $p$  must be zero. This mode is typically attributed to Jupiter, leading to the constraint  $g_5 = 0$ .

When analyzing the frequency spectrum of Pluto's  $h$ , Applegate et al. observed a peak near 0 (denoted line 'S') whose value was approximately equal to  $2p_1 - g_8$ , where  $p_1$  corresponds to the strongest frequency in both Pluto's  $h$  and  $p$  spectra and  $g_8$  is the vertical eigenfrequency due to Neptune. Because Pluto is currently in a Kozai resonance characterized by the libration of its argument of perihelion ( $\langle \dot{\omega} \rangle = 0$ ), we have  $\varpi = \Omega + \omega \Rightarrow \langle \dot{\varpi} \rangle = \langle \dot{\Omega} \rangle + \langle \dot{\omega} \rangle = \langle \dot{\Omega} \rangle$ , which implies that the eigenfrequencies of  $h$  and  $p$  are identical. Supposing that the frequency difference  $2p_1 - g_8$  either is or once was exactly zero, a secular resonance of the form  $2p_1 - g_8 - g_5 = 0$  (roughly corresponding to  $2\dot{\Omega}_P - \dot{\Omega}_N \approx 0$ ) becomes a likely candidate for generating a large change in Pluto's inclination.

### III. TECHNIQUES

To study the effects of orbital resonances numerically, we employed the integration engines HNBODY and HNDrag [8]. In addition to providing an efficient symplectic  $N$ -body integrator, the latter code allowed us to place artificial drag forces on each planet, simulating the migratory effects of scattering planetesimals. All integrations used a step size of 1 yr, ensuring a minimum of 10 steps per revolution for the closest body (Jupiter, when studying the full, modern Solar System).

We searched for mean motion resonances by plotting the appropriate resonant arguments (working modulo  $360^\circ$  for angular elements). Secular resonances, on the other hand, are less visible in the time domain, especially if they involve eigenfrequencies of the linearized system. We therefore applied a fast Fourier transform to the variables  $h$  and  $p$  in order to study their composition in the frequency domain. The definitions of these variables assume a system whose total angular momentum is entirely perpendicular to the reference plane. For systems with two massive non-central bodies whose longitudes of the ascending nodes are separated by  $180^\circ$ , this implies that

$$L_{xy} = M_1 \sqrt{a_1(1 - e_1^2)} \sin(i_1) - M_2 \sqrt{a_2(1 - e_2^2)} \sin(i_2) = 0 \quad (2)$$

We achieved a desired inclination difference  $\Delta i = i_2 - i_1$  by rearranging Equation 2 into the form

$$\frac{\sin(i_1)}{\sin(i_1 + \Delta i)} = \frac{M_2}{M_1} \sqrt{\frac{a_2(1 - e_2^2)}{a_1(1 - e_1^2)}} \quad (3)$$

and numerically solving for  $i_1$  using the van Wijngaarden-Dekker-Brent method [6]. For larger systems, inclinations were kept as small as possible for bodies other than Neptune, keeping orthogonal components of the total angular momentum minimal. However, this small deviation will affect the observed eigenfrequencies  $f_j$  and  $g_j$ , and in particular  $g_5$  may no longer be forced to vanish.

A Fourier transform assumes the data to be periodic over the interval being transformed. For physical data, however, this is rarely the case, and features in the resulting power spectrum can be lost due to spectral leakage. Mathematically, it is as if a square (“boxcar”) window has been applied to the data, zeroing out points beyond edges of the interval. Consequently, the Fourier transform of this square window is convolved with the desired spectrum, masking weaker peaks. Scaling the data by a window function that (typically) approaches zero at the boundaries of the interval drastically reduces these effects, as shown in Figure 1. Thus, in order to reduce spectral leakage, we applied a Hann window to  $h$  and  $p$  prior to taking the transform (this technique was also employed by Applegate et al. [2]).

The particular Hann window used is defined by

$$w_j = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi j}{M} \right) \right] \quad (4)$$

where  $w_j$  is the amount by which the  $j$ th data element is scaled. The act of windowing deweights data at the edges of the interval, increasing the variance of the power spectrum estimate. Overlapping data segments [6] provides a solution to this problem, but at the expense of resolution. This

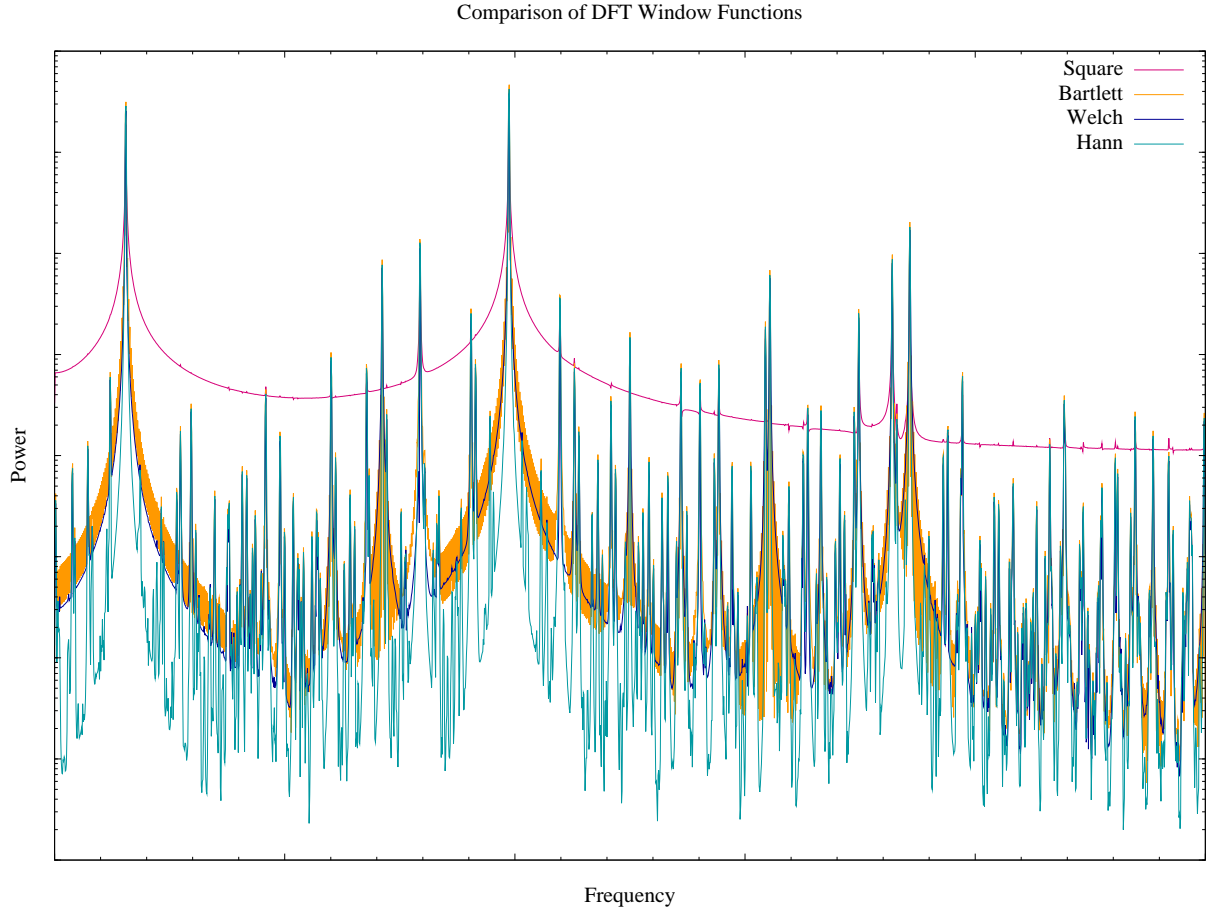


FIG. 1: A comparison of the effects of window functions on the discrete Fourier transform (DFT) of Saturn’s  $h$  in the Jupiter-Saturn system. The implicit square window masks many secondary peaks in a background several orders of magnitude higher than the others. Both the Welch and Hann windows represent low-power frequencies much more completely, with the Hann window featuring the narrowest peaks. The spectrum of the Bartlett window in this case oscillates sharply from one frequency to the next, possibly indicating an implementation error.

resolution is only recovered by increasing the length of the simulation, which is computationally expensive and makes the results more vulnerable to numerical errors and orbital instabilities. For this reason we chose not to overlap data segments when computing Fourier transforms. In addition, to further improve spectral resolution while avoiding these risks, we used inverse quadratic interpolation to fit a parabola through each peak, taking the location of its vertex as the center of the peak.

Due to storage constraints, the results of each simulation were sampled at rates on the order of

1000 yr. An effect of this sampling is that frequencies in  $h$  and  $p$  higher than the Nyquist frequency (equal to half the sampling rate) are aliased into the power spectrum, creating spurious peaks. The solution used by Applegate et al. [2]) is to sample each simulation at several different rates and to remove any peaks not present in all of the resulting spectra. Having noticed aliased peaks in some of our simulations, we automated a variant of this procedure by sampling each simulation at seven different rates, taking the Fourier transforms of  $h$  and  $p$  for each rate, interpolating the spectra onto a common set of abscissa, and taking the minimum value at each point. An example of the results for only three sampling rates is shown in Figure 2, where several secondary peaks present in individual spectra are found to be false. However, for systems with more than two significant non-central bodies, the relevant eigenfrequencies were easily identified, so we opted not to perform the above antialiasing procedure in such cases.

#### IV. MEAN MOTION RESONANCES

Pluto and Neptune are locked in several resonances, the most important of which is a 3:2 mean motion resonance with the condition  $3n_P - 2n_N - \dot{\varpi}_P = 0$  (where  $n \approx \dot{\lambda}$ ). As Neptune migrated outward via the preferential scattering of planetesimals, the location of this resonance swept across Pluto’s orbit, trapping Pluto and causing it to migrate with Neptune. During this time, Pluto’s eccentricity grew proportional to  $\sqrt{t}$  due to the presence of  $\varpi_P$  in the resonant argument. This is the widely accepted explanation for the origin of Pluto’s eccentric orbit [7].

Near this resonance lie several higher order variants which, by their dependence on  $\Omega_P$ , could potentially transfer energy from Neptune’s migration to Pluto’s inclination. The second-order candidates include  $6n_P - 4n_N - 2\dot{\Omega}_P = 0$  and  $6n_P - 4n_N - \dot{\Omega}_P - \dot{\Omega}_N = 0$ . Because the strengths of these resonances are proportional to  $i_P^2$  and  $i_P i_N$  respectively, their effects should be much weaker than the first-order  $e_P$  resonance. However, if these resonances are separated from the above resonance by a distance large compared to its breadth, Pluto may become trapped in one prior to reaching the eccentricity resonance. Alternately, as Pluto’s eccentricity rises, conditions may develop trapping Pluto in an inclination resonance from within its eccentricity resonance.

Early in the Solar System’s formation, Pluto’s elements were not constrained by the Kozai resonance ( $\dot{\Omega}_P = \dot{\varpi}_P$ ), so the difference between  $\dot{\Omega}_P$  and  $\dot{\varpi}_P$  serves to separate a 6:4 resonance from the 3:2 resonance. Our simulations of a simplified Neptune+Pluto system indicated that Pluto would likely cross this 6:4 resonance prior to being captured in the 3:2 resonance. Unfortunately,

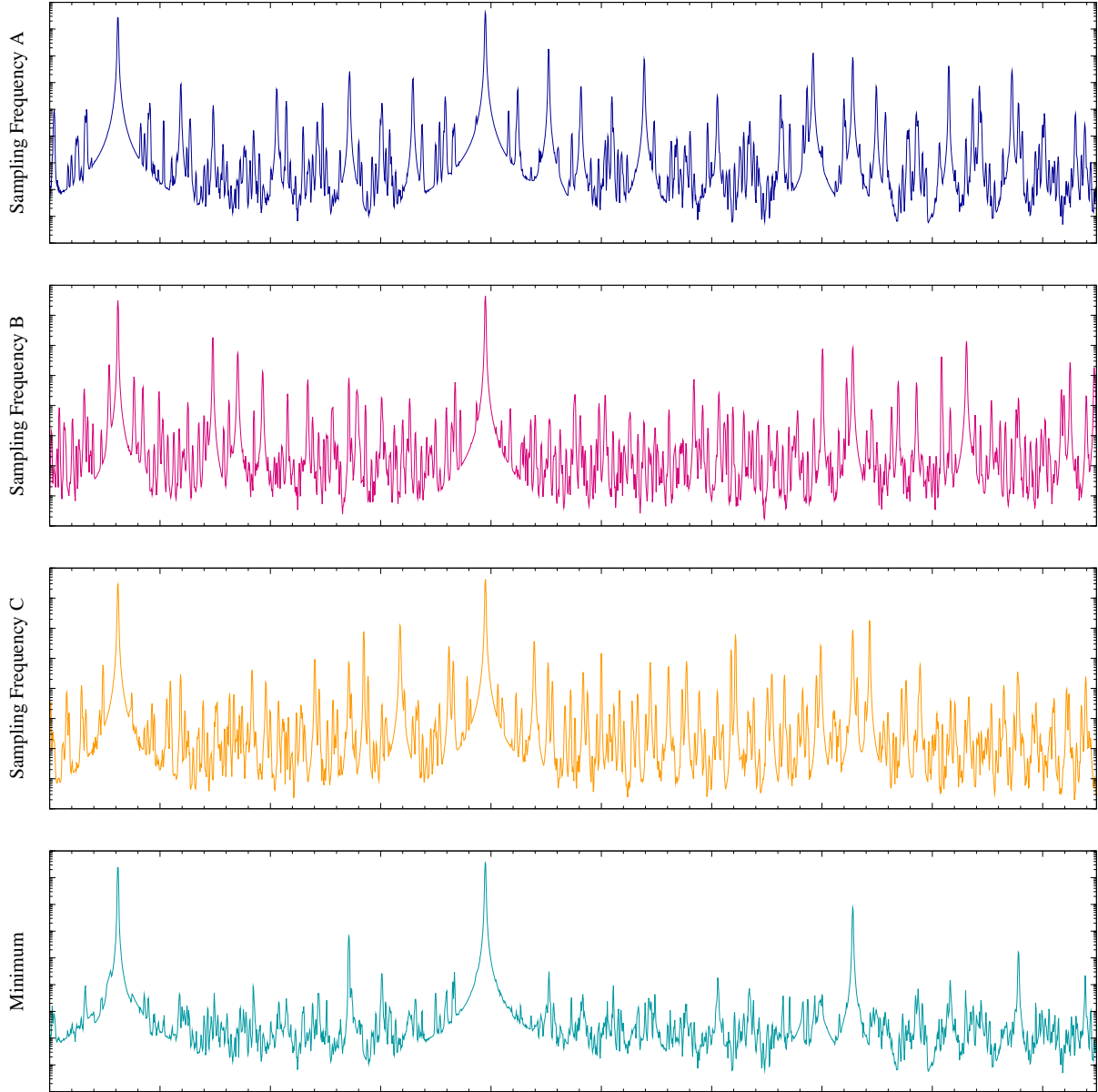


FIG. 2: Removal of aliased peaks in the discrete Fourier transform of Saturn’s  $h$  in the Jupiter-Saturn system. To check our implementation and interpretation of this transform, we compared the locations of the two dominant peaks with the predicted values of  $f_5$  and  $f_6$  from linear theory [5] and found them to match to within 15% (the peak in the  $p$  spectrum matched  $g_6$  to within 5%). The deviations are likely due to the near 5:2 mean motion resonance in the Jupiter-Saturn system which is not accounted for in the linear theory.



the resonance was so weak that Pluto’s inclination was only affected by tiny fractions of a degree, and capture into the resonance was infeasible at realistic drag rates (even these tiny results required migrating at a mere  $10^{-12}$  AU·yr<sup>-1</sup>).

The weakness of this resonance makes it difficult to identify, and in our simulations, we find no hint of an effect on Pluto’s inclination. Several simulations at very slow drag rates, however, did capture into an inclination resonance from within the eccentricity resonance. Upon closer examination, though, the resonance responsible turned out to be a secular resonance, which we discuss below. Based on the results of our simulations of a Neptune+Pluto system, a 6:4 mean motion resonance cannot explain Pluto’s high inclination, and the fact that it is currently in such a resonance is merely the product of being trapped in a Kozai resonance.

## V. SECULAR RESONANCES

Our principal goal was to examine the potential effects of a resonance between the nodal precession frequencies of Neptune and Pluto. Applegate et al. [2] noted that the fundamental frequency  $p_1$  in the spectrum of Pluto’s  $h$  was nearly half the value of  $g_8$ . We considered the possibility that  $2p_1$  is *exactly* equal to  $g_8$  and that the two planets are trapped in a secular resonance maintaining that relationship. As this resonance would involve  $\Omega_P$ , it could potentially have a long-term effect on Pluto’s inclination, explaining its present value of  $17^\circ$ .

We first encountered the effects of secular resonances while we were still examining the behaviors of mean motion resonances between Neptune and Pluto. In simulations with extremely slow drag rates ( $2 \times 10^{-11}$  AU·yr<sup>-1</sup>) and non-zero initial eccentricities ( $e_P = 0.02$ ), Pluto would occasionally become trapped in an inclination resonance, transferring energy from the libration of its eccentricity to its inclination. This is an example of a “secondary resonance.” Once the amplitude of Pluto’s eccentricity libration was small enough, its inclination would cease to grow while it continued to boost its eccentricity, now with a much smaller amplitude of libration. Such a scenario is plotted in Figure 3.

The source of this inclination growth may be a 1:1 secondary secular resonance  $\dot{\Omega}_P - \dot{\psi} = 0$ , where  $\psi$  is a libration phase. Regardless, in every scenario of this type Pluto’s inclination ceased to rise at a value far short of the required  $17^\circ$ . Though a larger increase in  $i$  could potentially be obtained when a larger initial value of  $e_P$ , the required drag rates are far too slow to have been present for this length of time during the Solar System’s formation, preventing this class of

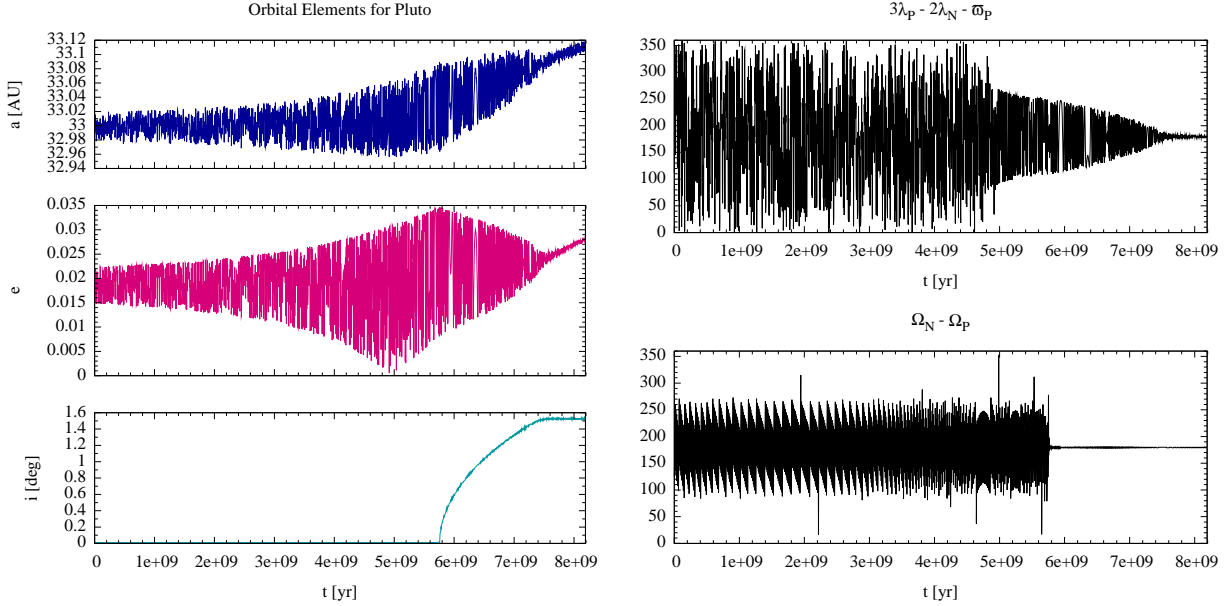


FIG. 3: Pluto’s semi-major axis  $a$ , eccentricity  $e$ , and inclination  $i$  over a 10 Gyr outward migration of 0.15 AU, along with relevant resonant arguments. The difference in the longitudes of the ascending nodes is forced to be zero by conservation of angular momentum, so any non-flat behavior in that resonant argument is a result of those orbital elements being poorly defined when  $i \approx 0$ .

behavior from being a viable explanation for Pluto’s current inclined orbit.

In order to directly target a classic secular resonance between  $\Omega_P$  and  $\Omega_N$ , we require at least one external massive body to impart various precession frequencies on Neptune and Pluto. We therefore considered several simple systems augmented with a Jupiter-like planet of even greater mass. By altering the mass and semi-major axis of this body, we could alter the precession frequencies of Neptune and Pluto’s nodes, as identified in the spectra of their respective  $ps$ . Changing Pluto’s semi-major axis allowed us to set the ratio of its precession frequency with respect to Neptune’s, but made it difficult to incorporate the important 3:2 mean motion resonance.

We first considered the 1:1 resonance  $\dot{\Omega}_P - \dot{\Omega}_N = 0$  as it should be stronger than the 2:1 resonance, and its resonant argument does not depend on an unknown frequency. Even without migrating through this resonance to achieve a capture or jump, Pluto’s inclination is severely affected by its proximity. Oscillations by as much as  $25^\circ$  were observed in a region at least 3 AU wide (Figure 4). Should Pluto be near this resonance when it was captured by Neptune, then depending on the phase of  $i_P$  in its oscillating cycle, Pluto could acquire a high inclination.

When we returned focus to full early outer Solar System, we found that the initial locations

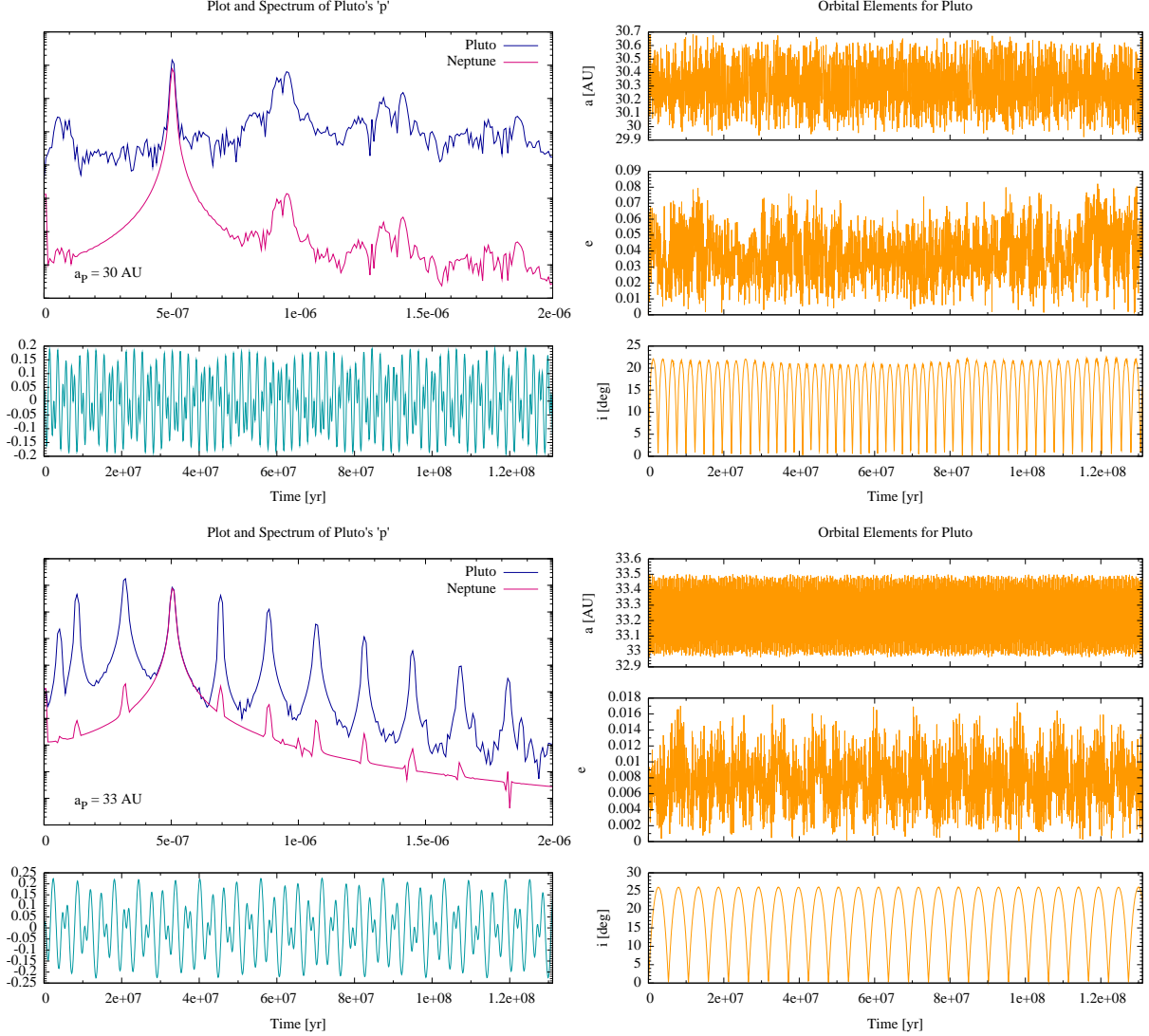


FIG. 4: Static effects of a 1:1 secular resonance within a 3 AU region. Precession on both bodies is induced by a massive Jupiter-like third body.

of the planets prior to migration places Pluto near enough to this same 1:1 secular resonance to have its inclination boosted by as much as  $10^\circ$  before becoming locked into its 3:2 mean motion resonance with Neptune (Figure 5). A plot of the relevant resonant arguments confirms the cause of the inclination jump in addition to capture in the proper 3:2 mean motion resonance. The final orbit is relatively unstable, but this scenario demonstrates that even without capture, a 1:1 secular resonance in the early solar system is potentially powerful enough to explain Pluto's current inclination.

In order to study the strength of a 2:1 resonance of the form  $2\dot{\Omega}_P - \dot{\Omega}_N - \dot{\Omega}_* = 0$  (where  $\dot{\Omega}_*$

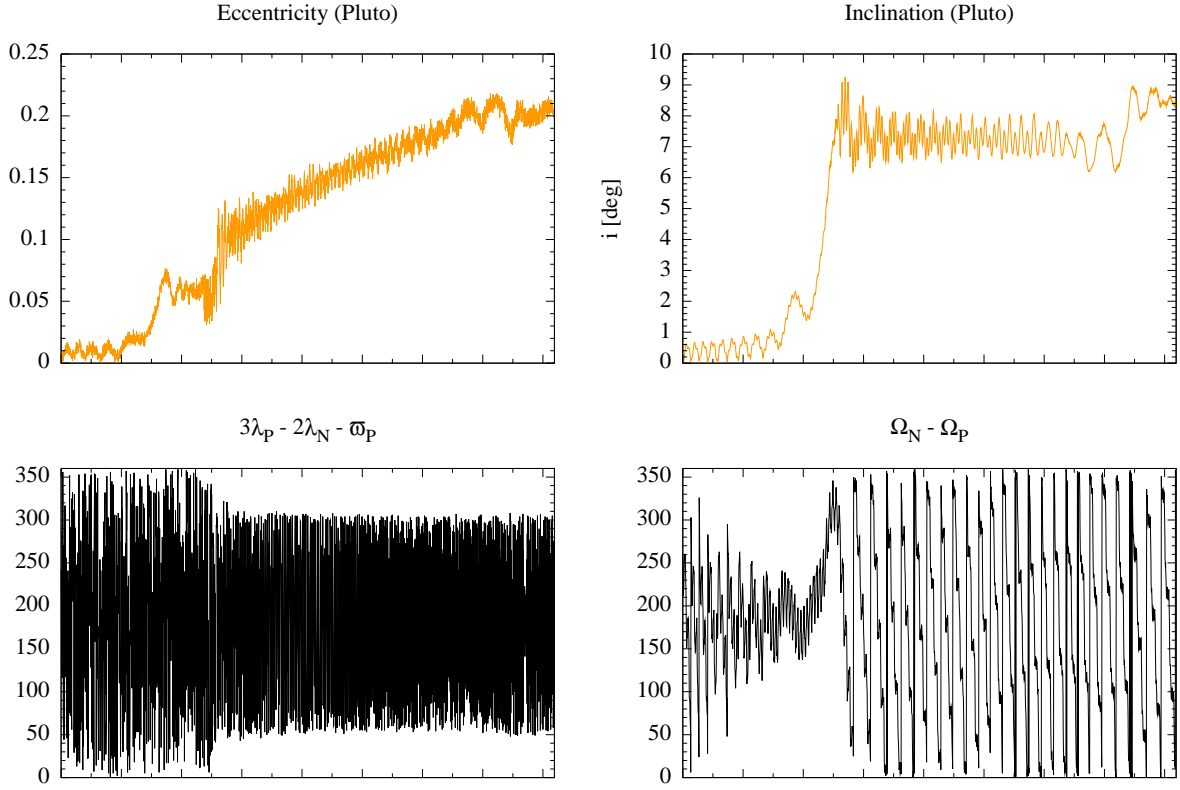


FIG. 5: Proximity to the resonance  $\dot{\Omega}_P - \dot{\Omega}_N = 0$  boosts inclination by nearly  $10^\circ$  but does not capture. In this case traversal of the resonance appears to be in the direction leading to strong kicks in inclinations.

is some slow, unknown rate), as we believe would be implied by the condition  $2p_1 - g_8 = 0$ , we artificially raised the mass of Uranus until this condition was met for a Pluto that had migrated to about  $a_P \approx 38$  AU (with  $e_P \approx 0.2$ ). This was achieved with  $M_U \rightarrow 1.8M_U$ . We then applied an artificial drag force to Pluto in order to move it through this region in both directions. While we do not have this kind of freedom with the real Solar System, the effects of both of these operations can likely be reproduced with more physically justifiable modifications. The effects of Uranus's greater mass on Neptune and Pluto should mimic those of increasing its semi-major axis prior to migration. Furthermore, the effects of a drag force on Pluto should mirror those of Neptune's migration in the opposite direction.

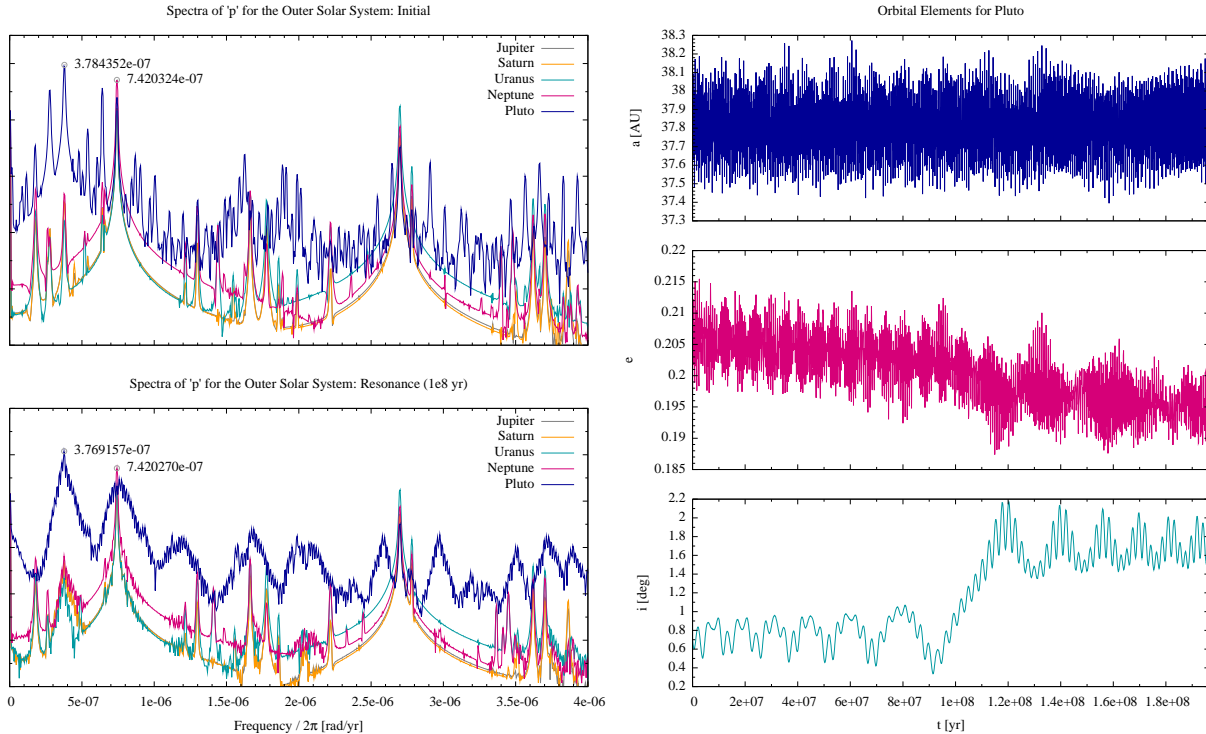


FIG. 6: A jump in Pluto’s inclination caused by drifting Pluto outward through a region where  $2p_1 - g_8 = 0$  at a rate of  $2 \times 10^{-8} \text{ AU}\cdot\text{yr}^{-1}$ . Uranus’s mass has been artificially increased ( $M_U \rightarrow 1.8M_U$ ) to achieve this resonance.

### A. Crossing of 2:1 Resonance with Decreasing $2p_1 - g_8$

With Pluto moving away from Neptune, we observe a jump in its inclination of about  $2^\circ$  as it crosses the proposed resonance (Figure 6). Initially,  $2p_1 \gtrsim g_8$ , and by distancing Pluto from the other planets we decrease  $p_1$  and cause Pluto to pass through this resonance from right to left in frequency space. The magnitude of this jump was unchanged when doubling Neptune’s initial inclination, so it is unlikely that it can be made strong enough to account for Pluto’s present inclination of  $17^\circ$ .

Note that, despite the drag force, Pluto’s semimajor axis is nearly unchanged. This is due to Pluto’s capture into its 3:2 mean motion resonance with Neptune ( $3n_P - 2n_N - \dot{\omega}_P = 0$ ). The force instead goes into decreasing Pluto’s eccentricity, reversing the effect of this resonance when Neptune migrates outwards. Nevertheless, Pluto’s nodal precession period was altered, as evidenced by the initial and resonant spectra. Following the jump  $p_1$  actually returns to very near its initial frequency.

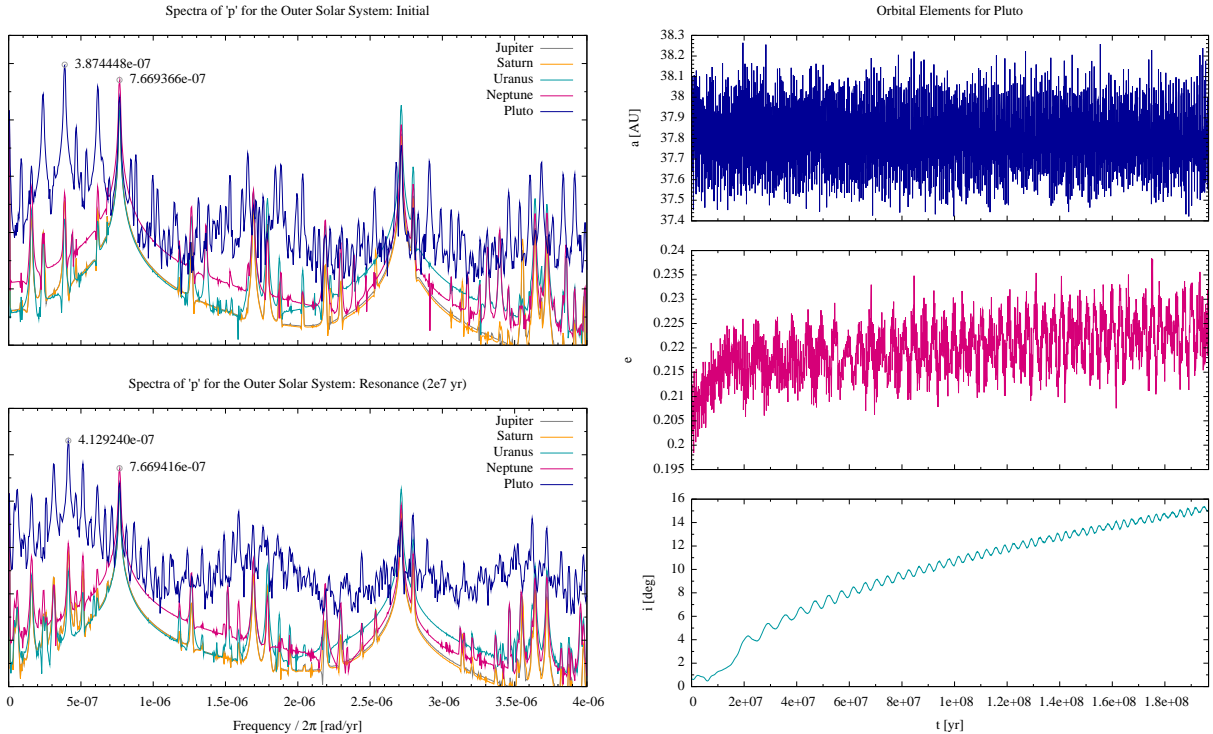


FIG. 7: Capture into an inclination resonance caused by drifting Pluto inward through a region where  $2p_1 - g_8 = 0$ . Uranus’s mass has been artificially increased ( $M_U \rightarrow 1.8M_U$ ) to achieve this resonance. Final conditions are extremely similar to Pluto’s present state.

### B. Crossing of 2:1 Resonance with Increasing $2p_1 - g_8$

By changing the sign of the drag force, we simulated Pluto drifting through the region surrounding the proposed resonance moving towards Neptune. As expected, changing the direction in which Pluto crossed the resonance caused it to become captured rather than experiencing a jump (Figure 7). The direction of this crossing reflects a scenario in which  $2p_1 \lesssim g_8$  initially, but a relative decrease in  $g_8$  brings the two values nearer to equality. This is what we would expect in the real Solar System as Neptune and Pluto migrate together. Being closer to the three perturbing bodies, Neptune’s precession frequencies would drop more rapidly than Pluto’s while the two migrated outward at the same rate. Therefore, current models for Solar System formation do not rule out a capture into this resonance due to directional considerations. Once again, the 3:2 mean motion resonance with Neptune prevents  $a_P$  from changing in response to the drag force.

Plotting the partial resonant argument  $2\Omega_P - \Omega_N$  for this system shows that the corresponding precession rates are nearly commensurate at about  $t = 10^8$  yr (Figure 8). If this is indeed the

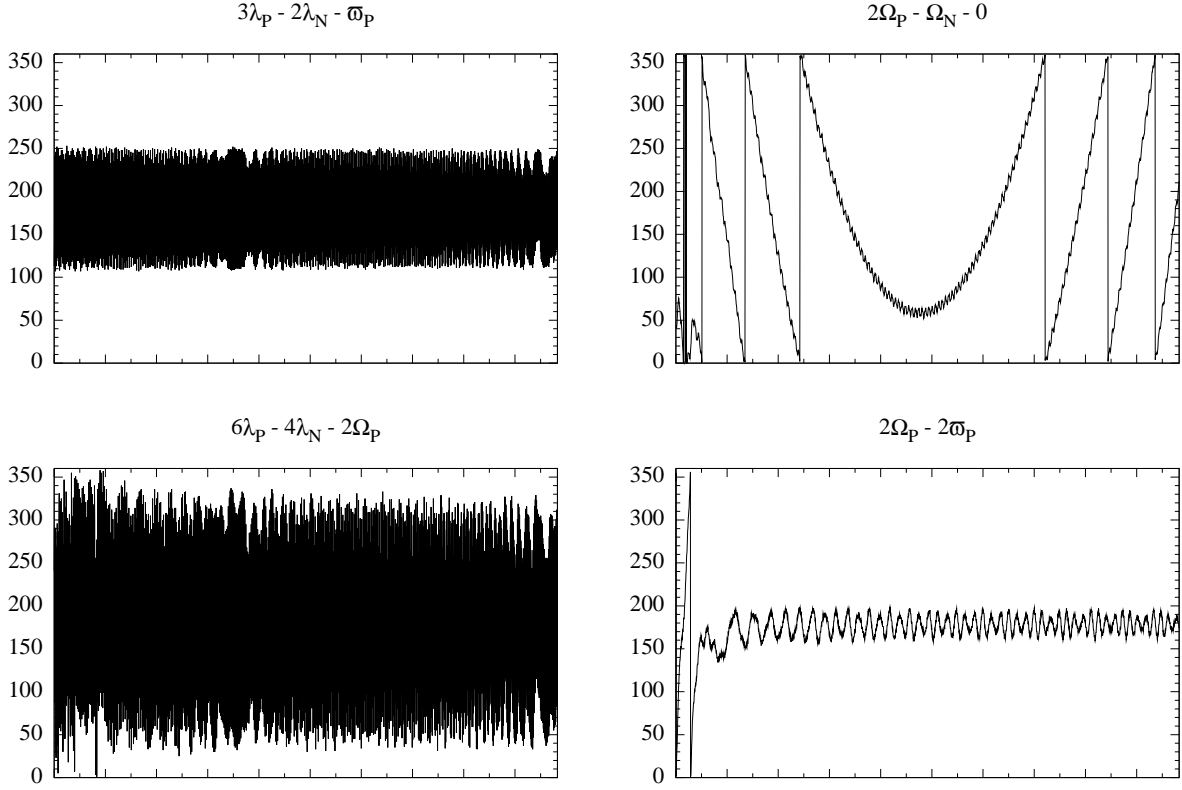


FIG. 8: Relevant resonant arguments for Pluto while trapped in an inclination resonance attributed to  $2p_1 - g_8 \approx 0$ . Uranus’s mass has been increased ( $M_U \rightarrow 1.8M_U$ ) to achieve this resonance. The drag rate on Pluto is  $-5 \times 10^{-8} \text{ AU}\cdot\text{yr}^{-1}$ , though the 3:2 mean motion resonance prevents  $a_P$  from feeling its effects.

resonance responsible for this behavior, then the missing frequency must be extremely slow. We suspect that the true resonance involves the eigenfrequencies  $g_8$  and  $g_9 (= p_1)$  rather than the node rates  $\dot{\Omega}_P$  and  $\dot{\Omega}_N$ . In this case, we could choose  $g_5$  as the missing frequency, which is always zero to conserve angular momentum, forming the key argument is  $2g_9 - g_8 - g_5$ . Another possibility is that the libration of Pluto’s argument of perihelion (the Kozai resonance) causes the behavior of Pluto’s inclination. We see that this resonance is active in the plot of  $2\Omega_P - 2\omega_P$  (confirming that  $f_9 = g_9 = p_1$ ), which guarantees libration of the argument  $6\lambda_P - 4\lambda_N - 2\Omega_P$  while in the 3:2 resonance  $3n_P - 2n_N - \dot{\omega}_P = 0$ . To distinguish between these two possibilities, we intend to run a set of similar simulations in the future in which  $2p_1 - g_8 \neq 0$ .

## VI. CONCLUSIONS

Judging from the results of these numerical simulations, secular resonances between Neptune and Pluto are worthy candidates for potentially explaining Pluto's high inclination. Both a nearby 1:1 resonance at the time of planet formation and a 2:1 resonance crossed during migration appear to be feasible without radically altering our models for Solar System formation and evolution. Furthermore, the 1:1 resonance is potentially strong enough to boost early inclinations by the required amount, while the 2:1 may be capable of capture and long-term inclination growth.

Future work is needed to determine exactly how these two scenarios constrain Solar System formation and to analyze the effects of either of these scenarios on the long-term stability of Pluto. The importance of similar resonances to other Kuiper Belt objects should also be considered.

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