1 Preface

This lecture notes are based on a PITP course given by Raman Sundrum. If you have any corrections please let me know at ajd268@cornell.edu.

2 Introduction

In this era we are stuck in two sets,

We focus our attention on the intersection of observability and plausibility, but we constantly wander back and forth between observability and plausibility to better understand what we should probe.

Strong dynamics is the blind spot of any vision of particle physics. this is because strong dynamics is very difficult to compute. In a sense you can say a weakly coupled theory what you write in a Lagrangian is what you get at in an experiment. In a strongly coupled theory this is not the case. There is a huge reprocessing from what you see in
the Lagrangian is what you actually measure. For example in QCD the Lagrangian looks
deevingly simple, but what comes out is the entire nuclear world!

When thinking about what is possible we tend to think much more about weakly
coupled additions to the SM then strong additions. This is of course because its much
easier to make predictions in weakly coupled theories. However, this bias is not apriori
justified and but how should we go about thinking along this path? This will be the focus
of the second part of these lectures.

3  Puzzles in the standard model and general relativ-
ity

The SM and GR have some well-known puzzles. Here we list some of things in no
particular order.

1. Dimensionless hierarchy: In the SM we have, \( g_{1,2,3}(m_Z) \), \( \lambda_H(m_Z) \sim 0.1 \). But in the
   Yukawa couplings we see hierarchy
2. Dimensionful hierarchy: \( G_F \gg G_N \)
3. Dark matter is not part of the SM
4. SM does not help us understand why \( \rho_{\text{baryons}} \sim \rho_{\text{DM}} \sim \rho_{\text{DE}} \)
5. \( \theta_{CP} \ll 1 \)
6. Couplings nearly meet at a high scale

3.1  Grand unification

[Q 1: Show plot of differential running for SM and MSSM] To explain the fact that
the gauge couplings all come together at some high scale we normally introduce grand
unification. However, there are some alternatives (see hep-ph/0212134). We do not
consider these here. The basic idea of grand unification theories (GUT) is if a set of
groups \( \mathcal{H}_i \) is part of larger broken group \( \mathcal{G} \) then we have at some scale,

\[
\mathcal{G} \to \prod_i \mathcal{H}_i
\]

and so the couplings are equal at the breaking scale.

Another nice property of grand unified theories is its influence on fermions. Fermions
are very special. Whatever fermions exist at some high energy theory, chiral symmetry
can protect their masses and they can drift down to low energies. In that sense fermions
are uniquely qualified as a kind of fossil of grand unification. Indeed we find that the SM
fermions fit beautifully into complete GUT multiplets.
In the SM case we need threshold corrections at the GUT scale that are about 20% of the differential running. This is not bad, but in the MSSM the threshold corrections are only about 4%. One of the reasons why strong dynamics is disfavored is because the MSSM running is so good looking. However, whether 4% is really that much better than 20% that any other possibility is wrong is uncertain. In some sense the SM is really what predicts unification, the MSSM is just a correction for this. We shouldn’t necessarily take it a sure signal that the MSSM is correct.

3.2 Yukawa hierarchy

There isn’t a perfect pattern mapped out by the Yukawa couplings, however there is some structure. If you take the hypothetical Yukawa matrix,

$$Y_{ij}^{U} \sim O(1)_{ij} \epsilon_i^q \epsilon_j^u$$  \hspace{1cm} (2)

$$Y_{ij}^{D} \sim O(1)_{ij} \epsilon_i^q \epsilon_j^d$$  \hspace{1cm} (3)

where the 9 $\epsilon$s are all $0 \leq |\epsilon_i| \leq 1$. By definition of first, second, and third generation we have,

$$\epsilon_1 < \epsilon_2 < \epsilon_3$$  \hspace{1cm} (4)

If you just set $y_{ttop} \sim 1$ this suggests that $\epsilon_u^3 \sim \epsilon_q^L \sim 1$ then you can use the data to solve for all the $\epsilon$s. For example,

$$V_{i \leq j}^{CKM} \sim V_{i > j}^{CKM} \sim O(\epsilon_i^q / \epsilon_j^q)$$  \hspace{1cm} (5)

Furthermore,

$$\epsilon_1^q \sim \theta_3^{cabbibo}, \epsilon_2^q \sim \theta_2^{cabbibo}, \epsilon_2^u \sim \frac{m_c}{m_t V_{ct}}$$  \hspace{1cm} (6)

You can get all these relations by what we know about the quark mixing angles and the quark masses. Its important to remember that flavor experiments probe energies way beyond what the LHC can probe. So to better understand the kinds of new physics that are allowed to need to also better understand flavor violation.

To see how this works consider some new physics mediator,

```
\begin{tikzcd}
& NP & \\
d \arrow[r] & s \arrow[l] & \bar{d} \\
& \bar{s} \arrow[u] & \arrow[u] \end{tikzcd}
```

We might want to imagine that the couplings to the SM are $O(1)$. However, this would produce very large $K\bar{K}$ mixing,
Such a process does occur in the SM but only through $W$-exchange, which is heavily suppressed. Therefore, the SM background is small and we are very sensitive to this. Assuming $\mathcal{O}(1)$ couplings constrains, $m_{NP} > 10^5\text{TeV}$. This is a very powerful filter when thinking about new physics.

Similarly, flavor diagonal couplings can also be sensitive due to CP violation. If we have,

$$d_L \quad d_R$$

$$\epsilon_R \quad \epsilon_L$$

If the couplings are $\mathcal{O}(1)$ and CP violating then they contribute to the anomalous dipole moment of atoms. The bound with this choice of couplings is $m_{NP} > 1000\text{TeV}$ (see e.g., hep-ph/0504231).

You might wonder whether people taking every new physics model and checking how it affects all the different types bounds. Thankfully there is an easy way to do this and that is using effective field theory (EFT). For example we can do,

$$\sim G_{NP}$$

The effective SM is given by,

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{n=5}^{\infty} G_{NP}^{-4} \mathcal{O}^{(n)} \quad (7)$$

Effective field theory is more complex then just “dropping the $p^2$”. A good reference on the subject is by Howard Georgi, “Weak Interactions”, and is available freely online.

In the SM we have,
So we commonly integrate out the weak scale physics to produce, $\mathcal{L}_{\text{eff}}^{(\text{lightSM})}$. If we have new physics at some higher scale,

Then we commonly build an effective theory for all the SM, $\mathcal{L}_{\text{eff}}^{(\text{SM})}$.

Its useful to have the “number one” cartoon of where the funny pattern of the Yukawa came from. While this paradigm may not be correct, its a nice pictorial idea that is also similar to other mechanisms. This mechanism works through introducing extra dimensions (XD).

The basic idea of XD is that we have $3 + 1$ infinite spacetime dimensions but then we add an additional small compactified dimension(s). This is shown pictorally below,

The different quantum fields occupy different modes in the extra dimensional “waveguide”. We would guess that the SM are the least wiggly in the extra dimension. This is because if you wiggle a lot then it costs you energy and we would have noticed this.

In general there is no reason to trust that the extradimensions are rectangular. In generally the wave functions in the extra dimensions can look like,
So there are all these wavefunctions which are rigid if you don’t put in a lot of energy, which correspond to the SM and maybe some additional particles that we are about to discover.

You may ask what is the wave function in higher dimensions. Its something like the Klien Gordan equation in higher dimensions,

\[(\partial_\mu^2 + \partial_{XD}^2 + m^2)\phi = \text{localized sources}\]  

(8)

If we imagine the fields are massless then we can write,

\[\phi = e^{ip_\mu x_\mu} \psi_{XD}\]  

(9)

For the SM \(p^2 \approx 0\). Then the \(\partial_\mu^2\) term vanishes. You see that the typical solution is an exponential except at special sources, the peaks of the mountains shown above.

What we’d like to know is if you take three particles in the SM we’d like to know their coupling in higher dimensions,

\[g_{ijk}^{SM} \sim O(1) \int dXD\psi_i \psi_j \psi_k\]  

(10)

with a lot of exponentials floating around you can easily pick up some hierarchical structure. But exponentials are very finicky. The integral will dominate where the exponentials are most peaked. So we can easily get a hierarchy between the masses due to exponentials in the extra dimensions. Larger couplings just means larger overlap in the extradimensions.

One may then ask why this doesn’t happen for the gauge bosons. If gauge bosons if the SM emerge from higher dimensional gauge fields then they would not have a mass term. This would make the exponentials much less sharply peaked and hence larger overlap with the Higgs wavefunction.
There are other ways to build the flavor structure of the SM, however, they are all similar to this idea here.

If new physics is of this form then even new physics is unlikely to be with $O(1)$ couplings and hence we should expect hierarchical couplings. There are some options to try to evade the flavor tests and continue to believe in new physics at accessible energies. We’d like to ask what kinds of hierarchical couplings are consistent with flavor tests, which are extremely stringent.

Another possibility if you are introducing new physics, don’t couple to the SM fermions. This is not completely gaurenteesing your safety but greatly reduces your risk. Another option to introduce more structure with some protective mechanism to avoid flavor violation. A classic example of this in supersymmetry is gauge mediated supersymmetry breaking.

### 3.3 Diquarks

Imagine a scalar, $\phi$ with the SM quantum numbers of a $u_R$ (e.g., “squark”). It can couple to quarks through,

$$\Delta \mathcal{L} = \lambda_{ij} \epsilon^{abc} \phi_a (d^i_R d^j_R c)$$

where, $a, b, c$ denote color indices, the Levi Cevita is required by gauge invariance, and $i, j$ denote flavor indices. The antisymmetry of the Levi Cevita kills the symmetric part of the $\lambda_{ij}$.

This thing is accidentally antisymmetric in flavor. What could of process can you get here? One possibility is,

$$\phi$$

Note that sign of the arrows here. This is a particle-particle interaction, not a particle antiparticle.

In fact if you just stuck to a two generation model you can still talk about flavor changing neutral currents (FCNC) but this cannot mediate one. In a two generation model, something antisymmetric in generation number is necessarily a flavor singlet. Thus it is incapable to produce a flavor changing neutral current.

You can compare this to adding in a second Higgs,

$$H'$$
This brings about dissasterous levels of FCNC. The three generational case also has a magical degree of safety, as long as you assume that the matrix, $\lambda_{ij}$ is hierarchical (that some elements are $\mathcal{O}(1)$ while other are small, with no requirement for which particle ones are small.

The TeVatron produced a few anomalies. The most well-known one being the forward backword top assymmetry. In the SM we have,

\[
\begin{align*}
\bar{q} & \quad \rightarrow \quad t \\
q & \quad \rightarrow \quad \bar{t}
\end{align*}
\]

which seems to insufficient to explain the TeVatron result. However, if we have a disquark then we have an additional diagram,

\[
\begin{align*}
q & \quad \rightarrow \quad \phi \rightarrow \quad \bar{t} \\
\bar{q} & \quad \rightarrow \quad t
\end{align*}
\]

To give a significant contribution this diagrams needs to couple most significant to the first and third generations, ($\lambda_{13}$). This seems like a crazy type of flavor structure but still have small diagonal elements. Nevertheless, this can be a significant contribution and still evading flavor bounds. So its important when trying to solve new physics anomalies to keep an open mind about the different possibilities that are out there.

4 Analogy

Lets consider the SM that emerges from some non-supersymmetric grand unification. In a GUT the $3 - 2 - 1$ gauge structure is enhanced to some bigger gauge group which contains new gauge bosons which mediate weird interactions,

\[
\begin{align*}
e^+ & \quad \rightarrow \quad X \rightarrow \quad u \\
d & \quad \rightarrow \quad u
\end{align*}
\]

The famous effect of such diagrams is that they allow the proton to the decay, $p \rightarrow \pi^0 e^+$. Lets take this interaction and consider the propagator correction to the positron,
The intermediate states that are possible are just baryons (in this case protons). So the positron (which cannot decay any further) is mixing with baryons. In this case you cannot any longer say that the mass eigenstate is just a lepton or just a baryon, its actually an admixture of the two. We have,

\[ |\text{exp.e}^+\rangle \sim |\text{elementarye}^+\rangle + \epsilon_p |p^{\text{comp}}\rangle + \ldots \] (12)

The experimental object is no longer elementary or composite, but some admixture of the two. You can think of the \( \epsilon_p \) as a mixing angle. By dimensional analyais the size of this angle is,

\[ \epsilon_p \sim \frac{\Lambda_{\text{QCD}}^2 m^2\chi}{m^2_X} \ll 1 \] (13)

This idea that the mass eigenstates doesn’t let you diagonalize “compositeness” is known as “partial compositeness”. To deal with this properly using effective field theory we need to integrate out the gauge boson and then find the running of the coupling down to the GeV scale through,

\[ \Delta \mathcal{L}(\text{GeV}) \rightarrow \frac{e u u d}{m^2_X} \exp\left( - \int_{\text{GeV}}^{m_X} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{4\pi} \right) \] (14)

where \( \frac{\alpha_s(\mu^2)}{4\pi} \) is the anomalous dimension (normally defined as \( \gamma \)). But, QCD is weakly coupling above a GeV. Thus the effect is going to be small since the theory is mostly perturbative. The strong interaction doesn’t have a significant effect on the result.

The magic of strong dynamics takes place over a large hierarchy in energies. This can happen when there is no renormalization group flow (the beta functions vanish),
Far from the fixed point you move quickly, however close to the fix point you move very slowly. So its possible that for a long hierarchy of scales you will remain in a strong coupling regime. At such a point $\alpha/4\pi$ is $\mathcal{O}(1)$ over a large hierarchy. Therefore, all of a sudden we’d have,

$$\epsilon \sim \frac{\Lambda_{\text{comp}}^2}{m_X^2} \left( \frac{m_X}{\Lambda_{\text{comp}}} \right)^\beta$$

(15)

where $\beta$ is known as a critical exponent in the language of condensed matter physics, which is determined by strong dynamics. If the critical exponent is $\mathcal{O}(1)$, this can have very interesting effects. Something that naively starts off being heavily suppressed by $m_X$, might be enhanced by $m_X$! In that case this interaction becomes more and more relevant in the IR. Then the separation between pure positron or pure proton breaks down completely and positron becomes part of the strong dynamics.

We want to explore the possibility that the Higgs boson that its part of strong dynamics. This is a classic idea created by Georgi and Kaplan. We now sketch how this works.

Suppose the fermions of the SM are elementary particles (like the positron) and they can only talk to the Higgs by mixing with the strong sector,

$$\langle H \rangle$$

Since we assume strong interactions we the elementary 3 body coupling will be roughly $4\pi$. However, we also need to include mixing angles to get the SM fermions themselves,

$$Y_{ij} \sim 4\pi \epsilon_i^u \epsilon_j^d$$

(16)
We call the “number of colors” in this strong dynamics as $N_{hyp}$. When $N_{hyp} \gg 1$ you can employ the large $N$ expansion. With this the coupling becomes $Y/\sqrt{N_{hyp}}$. The idea behind this expansion is that the interquark forces can be maximally strong but the interhadron forces can be parameterically weaker. In some sense, all the strong interactions are used up to make the hadron and residual interaction are parametrically weak and we can use that as our expansion parameter.

While we have no guarantee that $N_{hyp}$ will be large, we take this as one of the fundamental assumptions.

If we have a new strong interaction we should expect to have the SM fermions also couple to these new strong operators,

$$\mathcal{L} \supset \psi_{SM,i} O_{i}^{strong} \tag{17}$$

Since $O_{i}^{strong}$ are expected to carry SM quantum numbers we’d also expect the operators,

$$\mathcal{L} \supset A_{\mu}^{SM} J_{\mu}^{strong} \tag{18}$$

In the large $N$ approximation this tells us some of the kinds of states you can have,

$$O_{strong} |0\rangle = \text{mesons} \quad J_{\mu}^{strong} |0\rangle = \text{mesons} \tag{19}$$

We want to know if we are able to produce these composites. Since we are assuming the Higgs is a pseudo-goldstone boson of the theory [Q 3: What symmetry is being broken?], we’d expect it to be the lightest. But, how far away would the other composites be? We now make a quick estimate of the masses of the composites.

These new composites should show up as oblique corrections in electroweak precision. For example we must have,

$$W \rho_{hyp} W$$

This in particular affects the mass of the $W$ boson.

Another common example is the $Z \rightarrow b \bar{b}$ coupling.

The composite states coming in have the same quantum numbers as $b_L$, and this seems like a gauge boson which all it can see are gauge quantum numbers so no damage done.
But, this composite has a large coupling $4\pi/\sqrt{N}$ to turn into the Higgs which can turn into the vacuum. This flips $b_L$ into a $b_R$. This changes the coupling. The result is,

$$\frac{\delta g}{g} \sim \epsilon_{bL}^2 \frac{v^2}{\Lambda_{comp}^2}$$  \hspace{1cm} (20)

Since the top Yukawa is roughly 1 we must have,

$$1 \sim \frac{4\pi}{\sqrt{N}} \epsilon_{tR} \epsilon_{tL} = \frac{4\pi}{\sqrt{N}} \epsilon_{tR} \epsilon_{bL}$$  \hspace{1cm} (21)

where by electroweak symmetry we used $\epsilon_{tL} = \epsilon_{bL}$. Since we want a heavy top we expect, $\epsilon_{tR} \sim 1$, which implies that $\epsilon_{bL} \sim \frac{\sqrt{N}}{4\pi}$ and $\Lambda_{comp} \sim O(10\text{TeV})$.

All the EWP and flavor tests require $\Lambda_{comp} \gtrsim 20\text{TeV}$.

### 4.1 QCD + QED

Suppose we had a charged pion. We’d expect it to have electromagnetic interactions,

Knowing nothing about compositeness we’d expect to have,

$$\delta m^2 \sim \frac{e^2}{16\pi^2} \Lambda^2$$  \hspace{1cm} (22)

Then we’d philosophize what the cutoff ($\Lambda$) really means, its an intermediate step in the renormalized theory and its not physical, it goes away in dim reg, etc. Except in this context there is no discussion. Because the true picture is,

We can view this picture on the wavelength of the virtual photon. When the photon is soft, it cannot resolve all the detail and it looks like the first diagram and the result scales as $\Lambda^2$. But when the photon is short wavelength it doesn’t even see a scalar, but only a fermion. That means there is no quadratic divergence! So the true answer is not a quadratic divergence but some finite result given by,

$$\delta m^2 \sim \frac{e^2}{16\pi^2} \Lambda_{QCD}^2$$  \hspace{1cm} (23)

While, not obvious we can actually do better then that. We know how to sum the infinite number of gluons that affect the mass correction, but we omit its further discussion here.

Next, note that naturalness tells us that the mass corrections to the pion should be comparable to its mass. So naturalness suggests that $\Lambda_{QCD} \lesssim \frac{16\pi^2}{e^2} m_\pi$. This in fact predicts the maximum mass of the $\rho$ meson.
4.2 Back to new physics

Above we considered the pseudo-goldstones in ordinary QCD (the pions). We now return to the possibility that the Higg is a pseudo-goldstone boson.

We have the mass corrections,

\[ \delta m_{H}^2 = \frac{y_t^2}{16\pi^2} \lambda_{\text{comp}}^2 \]  

(24)

So we expect the new physics to show up at,

\[ \Lambda_{\text{comp}}^2 \sim 16\pi^2(125\text{GeV})^2 \sim (1.5\text{TeV})^2 \]  

(25)

[Q 4: Why did he say 0.5TeV]

5 Systematic expansion

In the strong force we can approach the physics using the quark model. However, we can make some very important predictions about the spectra using the chiral Lagrangian. There, all we need to assume is that chiral symmetry is conserved and all the physics follows from Lorentz invariance, unitarity, etc. To get the chiral Lagrangian all we need to do is assume a big gap between the pion and the \( \rho \) meson.

We’d like to repeat these steps for this new physics scenario. However, we don’t yet have the right small parameter to capture the physics of mesons. We have the small parameter \( 1/N \), however that’s not good enough [Q 5: why?].

Now lets consider the anomalous dimensions of different operators. Lets assume that operators that we have to have are separated in scale dimension by a gap which is called a scale dimension gap. In the renormalization group, when operators have very high dimension they rapidly become irrelevant. We can use the inverse of the size of this gap as a small parameter. Basically we assume the anomalous dimension “spectrum” to be of the form,
This is in fact one way of thinking of the Ads/CFT correspondance.

5.1 Unification

One concer of this setup is it could spoil unification. However, you can sort of keep the virtues of the SM unification by taking the new strong dynamics to cancel out of the differential running that determines whether you unify or not. This can be done by taking the flavor symmetry of the SM to be unified. This is discussed in detail in hep-ph/0502222.

It's not quite that you have to do,

\[
\lambda_{\text{comp}} + \ldots + \lambda_{\text{comp}} + \ldots
\]

Because the Higgs boson is actually part of the SM dynamics. Thus we should take it away from the SM contribution. Then the best fit to the data is to think of the right handed top quark as composite as well. In this case we also need to remove it from the non-composite contribution. Therefore we really have,

\[
\lambda_{\text{comp}} + \ldots + \lambda_{\text{comp}} + \ldots
\]

The composite diagrams don’t affect differential running [Q 6: why?] but the SM does. The running is now given by, [Q 7: Calculate running without Higgs and \( t_R \) or get from literature].

The overall running certainly depends on the strong dynamics! We expect the composite diagram goes as \( N_{\text{hyp}} \). If \( N_{\text{hyp}} \) is too big, then this running will drive you to a Lambda pole are not the leading story at all and it would be a disaster. Roughly we expect like to think of, \( N_{\text{hyp}} \sim \mathcal{O}(10) \) so \( 4\pi/\sqrt{N} \sim 3 \).
6 Phenomenology

There is an obvious set of composites that we want to see phenomenologically. They are the operators that mix with the SM elementary particles. The states,

\[ \mathcal{O}^{\text{comp}} |0\rangle \]

look like excited SM particles. In fact, in the extra dimensional geometric picture they are just described by the KK excitations of the SM particles. These states are narrow, i.e., bumps that we can seek at the LHC.

The details of the phenomenology are given in hep-ph/0612180, here we sketch the basic idea. We have,

\[ \bar{q} g \quad \text{KK gluon} \]

where the KK gluon is essentially a \( \rho \) meson. Even though the light fermions don’t mix very much with the KK gluon, you can still have such production through the gauge coupling.

The decays are always going to be to the heaviest fermions since they have the largest mixing with the SM. We can have decays to,

\[ t, b, \tau, W_L, Z_L, h \]

The top quark that comes as a decay product here,

\[ \bar{q} g \quad \text{KK gluon} \]

is not your grandfather’s top quark. In the TeVatron the top decay products would shoot in all the different angles. Because we are necessarily thinking about very massive resonances, the tops are highly boosted and their products are emitted in a narrow cone. Its a new challenge to tag such jets.