CRASH COURSE IN THE LHC

Lecture notes based on a lecture given by Nima Arkani-Hamed.
We discuss the tools to understand that plots released in
LHC measurements.

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I. INTRODUCTION

The goal of this discussion is to be able to figure out from plots of invariant mass, pT, counts, etc. to reconstruct the underlying theoretical model that exists. Further we want to see how a hadron collider differs from electron/positron collider as well as from Feynman diagrams.

II. LHC BASICS

The LHC is a proton-proton collider with a center of mass energy, $E_{CM} = 14\text{TeV}$. The Luminosity is $\mathcal{L} = 10^{33}\text{cm}^{-2}s^{-1}$ (low luminosity) or $\mathcal{L} = 10^{34}\text{cm}^{-2}s^{-1}$ (high luminosity). The Luminosity is related to the cross section, σ by the number of events per unit time:

$$\frac{\#\text{ events}}{\text{time}} = \sigma \times \mathcal{L} \tag{1}$$

Recall that in centimeters,

$$1\text{TeV}^{-1} \sim 10^{-17}\text{cm}^{-1}$$

Furthermore, typical strong interaction cross sections are

$$\sigma \sim \frac{\alpha_s^2}{\text{TeV}^2} \sim 10^{-36}\text{cm}^2 \tag{2}$$

Thus we get about an event a minute.

Most of the time when protons are sent flying towards one another they miss. However if you aim them just right and get lucky then the protons hit one another.

A proton is about 10^{-14}cm in radius. Thus the cross section for straight on collision is of order $\sigma \sim 10^{-28}\text{cm}$. These collisions produce debris right along the beam line. This is not what we are interested in. We want to study the short distance physics which produces particles at large angles. These are the heavier particles (hence short distance collisions) since they are not as strongly boosted (otherwise they'd be sent along the beamline.

For historical reasons we define

$$1\text{cm}^2 \equiv 10^{-24}\text{barn}$$

The LHC cross sections are of the order of a pb and fb. With $\sigma \sim pb$ we are talking about one event per minute at high luminosities. We summarize some different cross sections below

Cross section	Value	*Events*
σ^{tot}	$\sim$ (proton size)$^2 \sim 10mb$	10^9events/s
$\sigma^{t\bar{t}}$	$\sim 1000pb$	$10/s$
$\sigma^{W,Z}$	10^5pb	$10^3/s$
$\sigma^{WW} \sim \sigma^{ZZ}$	$100pb$	$1/s$

These are orders of magnitude larger then any beyond SM physics background. Wee have to beat these down by a factor of about a 1000 to get to signals of interest.

What makes it into the detector are sufficiently long lived particles. We see electrons, muons, Kaons, etc. We don't see particles such as the π^0 which decays very rapidly. For example the W decays very rapidly through

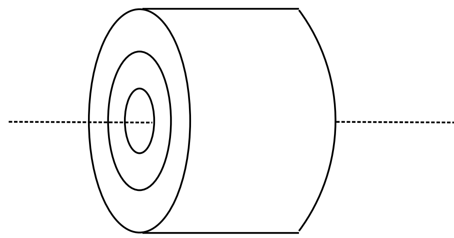
$$W \rightarrow \ell\nu$$

about 10% for each lepton. Since we have 3 leptons it is 30% "leptonic". We will emphasize leptons since they are easier to see then everything else at hadron colliders.

The Z decays through $f^+f^-, 3\%$ of the time and $\nu\nu, 6\%$ of the time. Thus we say it is leptonic about 10% of the time, invisible (we can't detect neutrinos) about 20% of thetime and hadronic, the other 70% of the time.

The top quark decays into a b, W and the W which each subsequently decay.

We will not go into details about the detectors However we note that deetectors have an onion like structure



It is an interesting irony that in order to test the smallest distances we need the largest detectors. The reason the detectors need to be so large is that the particles coming out have really high energies. We need to stop these particles. The only way to do that is to put as much material as you can in their way. In particular the muons are coming out with energies of order a few TeV.

The various layers of the "onion" are loosely shown below. Different particles show up in different places.

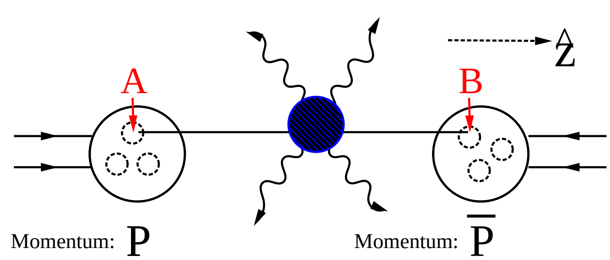
	Tracking Chamber	Electromagnetic Calorimeter	Hadronic Calorimeter	Muon Chamber
γ	✓			
e^+e^-	✓	✓		
$\pi^\pm, p$	✓		✓	
n			✓	

That's how you figure out which are the basically stable particles that come out of the collisions.

We did not mention some other special particles, the b, the c, and the t. To identify these particles special silicon detectors are added very close to the detector.

III. KINEMATICS

The LHC is a proton proton collider. People like linear colliders (electron-positron collider) since we can control everything. But linear colliders are for *sissies*. If you're at a linear collider and you're looking for a resonance at about 1000GeV but you're working at a center of mass energy of 1500GeV, then you will miss that resonance. That is because you may be able to control your states very easily, but that's also a problem since you get one initial state. However in a proton proton collider, the particles are messy bags of quarks and gluons. Each parton carries some fraction of the momentum of the proton. So you are actually scanning over a range of energies in every collision. Every collision is different!That's the good news. The bad news is that this is all happening simultaneously. So we don't control the initial state. We don't even know the center of mass frame that the collision happened in. We know the center of mass frame of the protons but not of the partons. This means that doing very precise things at hadron colliders is harder.



We define what's called lightcone coordinates:

$$p^\pm = p^0 \pm p^z \tag{3}$$
$$\mathbf{p}_T \tag{4}$$

$(\Rightarrow p^0 = \frac{1}{2}(p^+ + p^-), p^z = \frac{1}{2}(p^+ - p^-))$ and

$$\begin{aligned} A \cdot B &= A^0B^0 - A^zB^z - \mathbf{A}_T \cdot \mathbf{B}_T \\ &= \frac{1}{4}\left(A^+B^+A^-B^- + A^+B^- + B^+A^-\right) - \frac{1}{4}\left(A^+B^+A^-B^- - A^+B^- - B^+A^-\right) - \mathbf{A}_T \cdot \mathbf{B}_T \\ &= \frac{1}{2}\left(A^+B^- + A^-B^+\right) - \mathbf{A}_T \cdot \mathbf{B}_T \end{aligned} \tag{5}$$

Also throughout this lecture we will work in units such that the center of mass energy $(2E)$ is set to one. We label our vectors in the notation:

$$(p^+, p^-, p_x, p_y) \tag{6}$$

In these units we write

$$\begin{aligned} P &= (1, 0, 0, 0) \\ \bar{P} &= (0, 1, 0, 0) \end{aligned} \tag{7}$$

But the parton model tells us that we should think of the proton is being made out of a bunch of partons. We label the parton in the first proton as A and the parton in the second proton by B. We denote their fractional momenta as $f_{A/B}$. It's the parton collision that we are interested and will probe short distances.

One of the reason it's convenient to use these lightcone coordinates is we don't know the center of mass frame. These coordinates transform nicely under boosts in the z direction. We have

$$p^\pm \xrightarrow{\text{z boost}} \gamma(p^0 - \beta p^z) \pm \gamma(-\beta p^0 + 1)p^z \tag{8}$$

$$= \gamma(1 \mp \beta)p^0 \pm (1 \mp \beta)p^z \tag{9}$$

Now note that $e^\eta = \exp\left(-\sinh^{-1}(p_3/m)\right) = \gamma(1 - \beta)$. Hence

$$p^\pm \to e^{\pm\eta} p^\pm \tag{10}$$

We also have

$$\mathbf{p}_T \xrightarrow{\text{z boost}} \mathbf{p}_T \tag{11}$$

For this reason it is natural to define

$$p^\pm = \rho e^{\pm y} \tag{12}$$

for some y. Then under a boost we just have

$$y \to y + \eta \tag{13}$$

under a z boost and y is called the "rapidity". These coordinates are just the analogue to polar coodinates. The rapidity is just the analogue of the angle.

Now if we have a particle of mass m then $p^2 = m^2$. Now

$$\begin{aligned} p^2 &= p_0^2 - p_z^2 - \mathbf{p}_T^2 \\ &= p^+ p^- - \mathbf{p}_T^2 \\ &= \rho^2 - \mathbf{p}_T^2 \end{aligned} \tag{14}$$

Thus we have

$$\rho^2 = m^2 + \mathbf{p}_T^2 \tag{15}$$

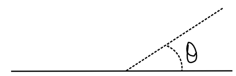
Notice the similarity of this equation with the energy equation, $E^2 = \mathbf{p}^2 + m^2$. This object is called for obvious reasons the transverse energy, E_T. This is the energy you would think an object would have if all you knew about it was it's energy. We can now also write

$$p^\pm = E_T e^{\pm y} = E_T(\cosh y \pm \sinh y) \tag{16}$$

or back in the original variables,

$$\begin{aligned} p^0 &= E_T \cosh y \\ p^z &= E_T \sinh y \end{aligned} \tag{17}$$

This is a measure of the angle relative to the beam that the particle is moving in. If $p_T \gg m$ then this is truly the angle. Keep in mind the following



$$\theta = 90^0, y = 0$$
$$\theta = 0^0, y \to \infty$$
$$\theta = 180^0, y \to -\infty$$
$$\theta = 10^0, y = 3.5$$

Finally in terms of these variables what is the Lorentz invariant phase space? One particle phase space is

$$d^4p\delta(p^2 - m^2) = dp^0dp^zd\mathbf{p}_T\delta(p^+p^- - (\mathbf{p}_T^2 + m^2)) \tag{18}$$

The Jacobian for the transformation of equation 3 is (the determinant of $\partial(p^0, p^z)/\partial(p^+, p^-)$).

$$\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2} \tag{19}$$

so we have

$$d^4p\delta(p^2 - m^2) = \frac{1}{2}dp^+dp^-d\mathbf{p}_T\delta(p^+p^- - (\mathbf{p}_T^2 + m^2)) \tag{20}$$

The Jacobian for expression 17 is (we let ρ denote a variable and $E_T^2 \equiv \mathbf{p}_T^2 + m^2$)

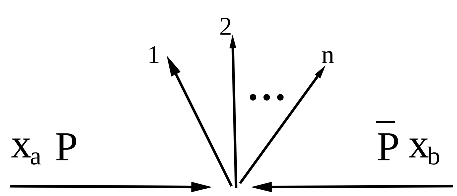
$$\begin{vmatrix} \cosh y & \rho\sinh y \\ \sinh y & \rho\cosh y \end{vmatrix} = \rho\cosh^2 y - \rho\sinh^2 y = \rho \tag{21}$$

So we have

$$d^4p\delta(p^2 - m^2) = \frac{1}{2}\rho d\rho dy d\mathbf{p}_T\delta(\rho^2 - E_T^2) \tag{22}$$
$$= \frac{1}{4}d\rho^2 dy d\mathbf{p}_T\delta(\rho^2 - E_T^2) \tag{23}$$
$$= \frac{1}{4}dy d\mathbf{p}_T \tag{24}$$

Note: When Nima derived this formula he did not have a factor of $\frac{1}{4}$. At the moment I am not able to figure out whether he was being sloppy or I made a mistake. For here on out I follow his notes and omit this factor

Recall the situation under consideration



We have

$$x_aP = (x_a, 0, 0, 0) \tag{25}$$
$$x_b\bar{P} = (0, x_b, 0, 0) \tag{26}$$
$$P_j = (E_je^{y_j}, E_je^{-y_j}, \mathbf{p}_{T,j}) \tag{27}$$

where P_j denotes the mometa of every outgoing particle, E_j is the transverse energy of any outgoing particles, and $\mathbf{p}_{T,j}$ is the transverse momenta of out outgoing particles.

So by conservation of momenta we have

$$x_a = \sum_j E_j e^{y_j} \tag{28}$$

$$x_b = \sum_j E_j e^{-y_j} \tag{29}$$

$$0 = \sum_j \mathbf{p}_{T,j} \tag{30}$$

Now there is something particularly nice about this. We don't know the energies of each collision. However if we could see all the particles that come out, then we could measure the momentum fractions using the rapidities and transverse energies. We will use this over and over again. We are going to convert things that depend on the x 's of the partons into things we can compute.

The center of mass of the collision is

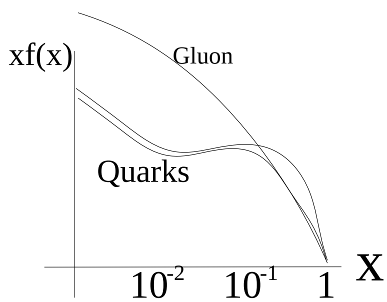
$$s_{a,b} = (x_a P + x_b \bar{P})^2 = 2x_a x_b P \cdot \bar{P} = x_a x_b \tag{31}$$

So the center of mass is just the product of the two x 's. So now lets talk about the parton model.

IV. PARTON MODEL

While we can't predict exactly what the x 's are for any particular collision, there is a certain probability for finding a with a given momentum fraction between $(x_a, x_a + dx_a)$ and it's given by $f(x_a)dx_a$. We can measure this (we call these Parton Distribution Functions or PDF)for instance as people did originally from an experiment from deep inelastic scattering (take an electron and collide it with a proton). It is a non-trivial fact that in a process independent way you can think of a proton as the same bag. In other words that the same PDFs can be used for all experiments. This true fact is called "factorization".

Lets take a quick look what these PDFs look like. These PDFs run like everything else. We take $Q^2 \sim \text{TeV}^2$. The most naive guess would be that since the proton is made of 2 up quarks and a down quark that the PDFs would be completely flat. This turns out not to be the case at all. The true graphs take the form



Notice that all the PDF's go to zero as $x \to 1$ since the probability that one parton will carry all the momentum is zero. As we'll see in a moment all the collisions at the LHC take place at about $x < 0.1$. That's why the LHC is essentially a gluon gluon collider.

So what the parton model says then is that

$$d\sigma = \int_0^1 dx_a dx_b f_a(x_a) f_b(x_b) \times \overbrace{\left(\frac{|\mathcal{M}|^2}{s_{a,b}}\right)}^{d\sigma_{ab}} \times \delta^{(4)}(x_a P + x_b \bar{P} - (p_1 + ... + p_n)) \times dy_1 d\mathbf{p}_{T,1} ... dy_n d\mathbf{p}_{T,n} \tag{32}$$

We know what x_a and x_b are in terms of y_j. We can do the integral and we have

$$d\sigma = f_a(x_a) f_b(x_b) \frac{|\mathcal{M}(ab \to 12...n)|^2}{x_a x_b} \times \prod_{j=1}^{n} dy_j d\mathbf{p}_j \delta(\sum \mathbf{p}_{T,j}) \tag{33}$$

where here $x_a = \sum_j E_j e^y$ and $x_b = \sum_j E_j e^{-y}$. Furthermore we used expression 31 for the center of mass energy.

Lets do the particular case of $n = 2$. Then

$$d\sigma = f_a(x_a)f_b(x_b)\frac{|\mathcal{M}(ab \to 12)|^2}{x_a x_b} \times (dy_1 dy_2 d\mathbf{p}_T) \tag{34}$$

where the integral over the $d\mathbf{p}_T$ of the second particle enforces $\mathbf{p}_{T,1} = -\mathbf{p}_{T,2} \equiv \mathbf{p}_T$.

We concentrate on the case for $2 \to 2$ scattering. We want to know the Mandlestam variables.

$$s = x_a x_b = E_1^2 + E_2^2 + 2E_1E_2\cosh(y_2 - y_1) \tag{35}$$
$$t = (x_a P - p_1)^2 = -(\mathbf{p}_T^2 + E_1E_2 e^{y_2 - y_1}) \tag{36}$$

(Exercise) These only depend on the tranverse energies and the difference in the rapidities.

Also $|\mathcal{M}|^2\Big|_{\text{spin ave}}$ only depends on s and t. So that motivates us in writing the phase space factor to use variables which are the average of y_1, y_2 and the difference of y_1, y_2. We write

$$dy_1 dy_2 d\mathbf{p}_T = d\bar{y}\, d\Delta y\, d\mathbf{p}_T \tag{37}$$

where $\bar{y} = (y_1 + y_2)/2$ is the average of the rapidities and $\Delta y = y_2 - y_1$ is the difference in the rapidities. Note that the Jacobian is

$$\begin{vmatrix} -1 & 1 \\ 1/2 & 1/2 \end{vmatrix} = \left| -\frac{1}{2} - \frac{1}{2} \right| = 1 \tag{38}$$

$\bar{y}$ not being zero means the center of mass frame is boosted with respect to the lab frame. A lot of what's done in this business is to write these variables in different forms. We can trade variables into any two independent variables:

$$(\Delta y, \mathbf{p}_T) \leftrightarrow (s, t) \leftrightarrow (s, \mathbf{p}_T) \tag{39}$$

We have

$$ds = 2E_1E_2\sinh(\Delta y)d\Delta y \tag{40}$$
$$= \underbrace{2E_1E_2\sinh\left(\cosh^{-1}(\alpha)\right)}_{1/\mathcal{J}} d\Delta y \tag{41}$$

where $\alpha = \frac{s - E_1^2 - E_2^2}{2E_1E_2}$. Simplifying we get

$$\mathcal{J} = \frac{1}{\sqrt{((E_1 - E_2)^2 - s)((E_1 + E_2)^2 - s)}} \tag{42}$$

$$d\bar{y}\, d\Delta y\, d\mathbf{p}_T = \mathcal{J}(s, \mathbf{p}_T)\, d\bar{y}\, ds\, d\mathbf{p}_T \tag{43}$$

When the particle masses are the same $M_1 = M_2$ so $E_1 = E_2 = E$. We get

$$x_a = \sqrt{s}e^{\bar{y}} \tag{44}$$
$$x_b = \sqrt{s}e^{-\bar{y}} \tag{45}$$

and

$$d\sigma = d\bar{y}\, ds\, d\mathbf{p}_T \mathcal{J}\frac{|\mathcal{M}(s, \mathbf{p}_T)|^2}{s} f_a(\sqrt{s}, \bar{y}), f_b(\sqrt{s}, \bar{y}) \tag{46}$$

where $\mathcal{J} = \frac{1}{\sqrt{s(s-4E^2)}}$

If we only measure transverse stuff but we don't measure $\bar{y}$ then we have

$$d\sigma = ds\, d\mathbf{p}_T \mathcal{J}(s, \mathbf{p}_T)\frac{|\mathcal{M}(s, \mathbf{p}_T)|^2}{s} \times \rho_{ab}(s) \tag{47}$$

where $\rho_{ab} = \int d\bar{y} f_a(\sqrt{s}e^{\bar{y}}) f_b(\sqrt{s}e^{-\bar{y}})$ is called the "Parton Luminosity". The fact that we can well approximate these f's by power laws. Due to the exponential dependence of these f's their product is approximately flat and does not vary with $\bar{y}$. Thus $\rho_{ab} \sim s^{-2}$. The Parton Luminosities fall as a large power to the energy!

V. PRODUCTION

Suppose we have

$$ab \to \text{resonance} \to 12$$

and $1, 2$ are some massless SM particles. In this case we know that the amplitude looks like a Breit Wigner resonance:

$$|\mathcal{M}|^2 \sim \frac{1}{(s-M)^2 + M^2\Gamma^2} \to \mathcal{C}\delta(s - M^2) \tag{48}$$

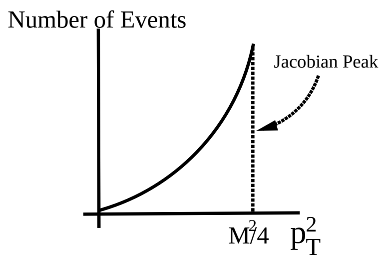
where M is the mass of the resonance.

Taking the formula for the cross sectioand integrating over s we get

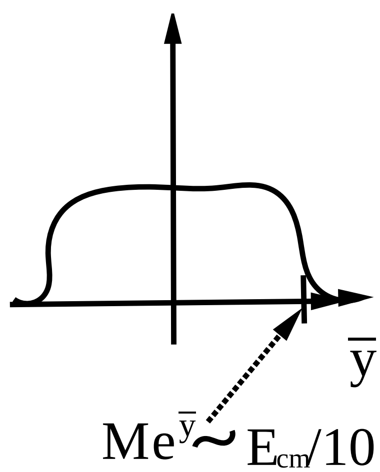
$$\frac{d\sigma}{\mathcal{C}} = d\mathbf{p}_T d\bar{y} \frac{\mathcal{J}}{M^2} f_a(Me^{\bar{y}}) f_b(Me^{-\bar{y}}) \tag{49}$$

$$= \underbrace{\left(\frac{d\mathbf{p}_T}{\sqrt{1 - \frac{4E^2}{M^2}M^2}}\right)}_{\mathbf{p}_T \text{ dependance}} \underbrace{\left(\int d\bar{y} f_a f_b\right)}_{\bar{y} \text{ dependence}} \tag{50}$$

We have



and



The total cross section (we just integrate the term above)

$$\sigma_{tot} \sim \frac{1}{M^2} \int d\bar{y} f_a f_b \tag{51}$$

$$\frac{1}{M^2} \rho_{ab}(M^2) \sim \begin{cases} \frac{1}{M^6} & gg \\ \frac{1}{M^{5.3}} & \bar{u}u, \bar{d}d \end{cases} \tag{52}$$

The cross sections drop like a crazy power of the mass!It drops like this power because of the PDFs. That's why at the LHC we have to be prepared to cover a huge range of cross sections. A factor of 10 in the mass corresponds to factor of 10 in the rate.

Lets now consider a different case:

$$ab \to 12 \tag{53}$$

where now 12 are of equal mass M. Then we have

$$d\sigma = d\bar{y}d\left(\mathbf{p}_T/M\right)\frac{ds/M^2}{\sqrt{\frac{s}{M^2}\left(\frac{s}{M^2}-\frac{4E^2}{M^2}\right)}}\frac{|\mathcal{M}|^2}{s/M^2}f_a f_b \tag{54}$$

$$= \frac{d\left(\frac{\mathbf{p}_T}{M}\right)d\left(\frac{s}{M^2}\right)}{\sqrt{\frac{s}{M^2}\left(\frac{s}{M^2}-\frac{4E^2}{M^2}\right)}}\frac{|\mathcal{M}|^2}{(s/M^2)}\rho_{ab} \tag{55}$$

The important point is that all these factors are falling with s. ρ_{ab} is falling as $1/s^2$. No amplitude that we care about will fall with s. However s can't be less then $4E_T^2$ scared due to the pole. Thus this expression is **dominated** by the smallest s and hence the smallest $\mathbf{p}_T$. The punch line is as follows

HEAVY PARTICLES ARE PRIMARILY PRODUCED AT THRESHOLD

At a 10TeV collider you may think that you would make highly boosted top quarks. However that is wrong!The tops produced are going to be almost at rest. No matter what the energy is, the heavy particles are almost essentially made at rest.

This is an incredibly useful fact. This does not occur for a e^+e^- collider. We have

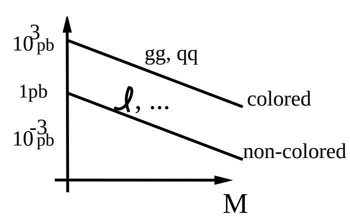
$$\left\langle \frac{p_T^2}{M^2} \right\rangle \sim \begin{cases} 1/3 & gg \\ 1/2 & \bar{u}u \end{cases}$$

and

$$\langle s \rangle \sim 5 - 6M^2$$

for $gg, \bar{u}u$. This gives you a nice measure of how boosted things are. The fact that these particles are created at threshold corresponds to very big qualitative difference between a linear collider and hadron collider. If you make a e^+e^- collider then the cross section goes as $\sigma \sim \frac{\alpha^2}{E_{CM}^2}$. There is nothing you can do about it. By contrast at a hadron collider the cross section goes up with the energy: $\sigma \sim \frac{1}{M^2}\left(\frac{E_{CM}}{M}\right)^4$. This also explain why there is such a jump from the Tevatron to the LHC. Not only do we have access to higher energy particles, we also make many more particles per second.

For example if you look at the cross section to make a SUSY particle.



Therefore the LHC makes colored objects up to about $\sim$ 5TeV and not colored particles up to about 300 $\sim$ GeV. In total there are many more colored particles produced. The excitement at a hadron collider is really big right in the beginning since that is when the big gain in energy will show itself.