DARK MATTER
LECTURE NOTES

Lecture notes are largely based on a lectures series given by Neil Weiner at PITP on dark matter

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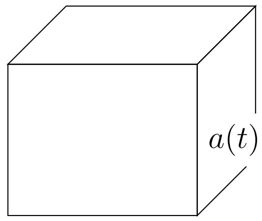
Contents

1 Preface

This lecture notes are based on a PITP course given by Neil Weiner . If you have any corrections please let me know at ajd268@cornell.edu.

2 Introduction

The universe is charactorized by a scale factor, $a(t)$. The way we imagine this is we have some box of the universe,



The universe is expanding which implies we have a charactoristic parameter,

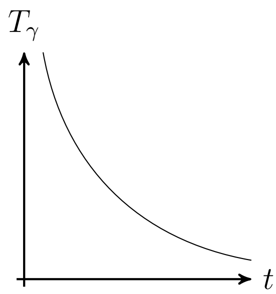
$$H \equiv \frac{\dot{a}}{a} \tag{1}$$

The way to think about the Hubble parameter is that H^{-1} is a charactoristic time for expansion. The Hubble time represents how long it take the universe to expand by an $\mathcal{O}(1)$ factor. Processes which happen on timescales, $t \ll H$, are happening in an effectively static universe while those which occur on $t \gtrsim H$ happen on timescales where the universe is expanding and it then has to be taken into account.

The universe is charactorized by temperature,

$$T_\gamma \sim a^{-1} \tag{2}$$

As the universe expands we have,



So there is a relationship between $t \leftrightarrow T^{-1}$ and we can use either one to describe the evolution of the universe. Its much easier to use temperature to describe its evolution since those are the units that is how we normally describe rates.

Fluids in the universe are generally given by "radiation" and "matter". Radiation (matter) is any relavistic (nonrelativistic) matter in the early universe.

The number density of any particle that does not have its number changed scales as,

$$n_{particle} \propto a^{-3} \tag{3}$$

This has two consequences. The first is that,

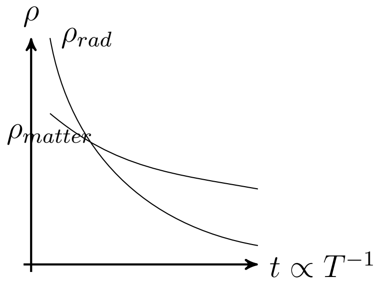
$$\rho_{rad} \propto a^{-4} \propto T^4 \tag{4}$$

The way to understand this is that the temperature of the radiation is dropping like a^{-1} and the number density, we already know falls like a^{-3}. [Q 1: Understand how this works better]

The energy density of non-relativistic matter is,

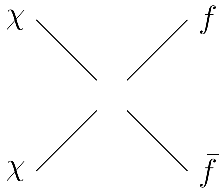
$$\rho_{matter} \approx a^{-3} \propto m_\chi n_\chi \propto m_\chi T^3 \tag{5}$$

So if we think about how different fluids behave as a function of the inverse temperature we have,



We refer to the period where $\rho_{rad} > \rho_{matter}$ as radiation dominated and when $\rho_{matter} > \rho_{rad}$ as matter dominated. These are the interesting periods which will be the focus of this lecture.

The above formula are for fluids with a conserved numbers. However, what about particles in equilibrium with other particles and can change their number densities? So you can imagine we have systems with some particle, χ, that interacts with the SM fermions, f, through,



So χ can convert into SM matter and back. If this process is in equilibrium then,

$$n_{rad} \sim T^3 \tag{6}$$

For non-relativistic matter,χ, thats heavier then f, there is another scale in the problem and we have,

$$n_{NR} \sim (m_\chi T)^{3/2} e^{-m_\chi/T} \quad \text{(with no chem. pot.)} \tag{7}$$

So non-relativistic matter very quickly depletes away due to the exponential decay.

The Hubble rate is related to the total energy density of the universe by,

$$H^2 = \frac{\rho_{tot}}{M_{Pl}^2} \sim \sqrt{g_*}\frac{T^2}{M_{Pl}} \tag{8}$$

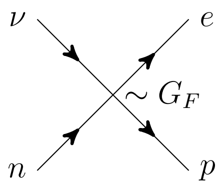
where g_* counts the number of degrees of freedom.

We denote the rate for the $\chi \to f$ process as Γ. The question we want to ask is,

$$\Gamma > H \quad ? \tag{9}$$

Recall that H^{-1} is the time it takes the universe to double in size. So if Γ is higher then that it means the process is happening faster then the universe is expanding. If $\Gamma \gg H$ then this process happens many times and by today and we say this process is equilibrium. [Q 2: How do you calculate a rate for a 2 body process?]

Lets now consider as example the process of neutrino-neutron to proton-electron conversion,



You have a process where if you just have a free neutron in the early universe flying around its going to eventually encounter a neutrino its going to scatter off of it and form an electron and proton. So a question you might ask is how long is this process going to stay in equilibrium. Neutrons are eventually going to decay and there is also a proton-neutron mass difference so is this process in equilibrium for a long time or not? We can easily estimate this. Naive dimensional analysis gives,

$$\Gamma \sim G_F^2 T^5 \tag{10}$$

We want to know when this rate is larger then $H \sim \frac{T^2}{M_{Pl}}$. Equivalently this is when,

$$G_F^2 T^5 > \frac{T^2}{M_{Pl}} \tag{11}$$

This occurs when,

$$T = \left(\frac{1}{M_{Pl} G_F^2} \right)^{1/3} \approx 0.9\text{MeV} \tag{12}$$

So this process will stop being at equilibrium at $t \approx T^{-1} \to (0.9\text{MeV})^{-1}$. Since this is close the proton-neutron mass difference this is one of those rare instances where you need to do a careful calculation. This also tells us that prior to this era protons and neutrons will be in equilibrium.

3 What we know about DM

Its essential to remember two facts about DM,

1. No one knows anything about DM
2. We already know a lot about DM

These two should be kept in mind whenever discussing DM. Point 1 refers theoretically to how a normaly DM candidate should act. Point 2 is experimentally how much we know from cosmilogical constraints. If we make a chart of what people have though over the course of time,

Time	Candidate
< 1930	none
70's-80's	Baryonic material (gas, unegnited stars, ...)
80's - 90's	Neutrinos
90's $<$	Neutralinos/axions

We write it in this form to emphasize that what a reasonable model of DM is changes over time and we shouldn't be prejudice toward one form or another.

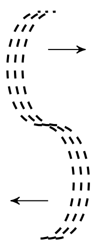
There are five most importantant pieces of evidence for DM,

1. Galaxies in clusters

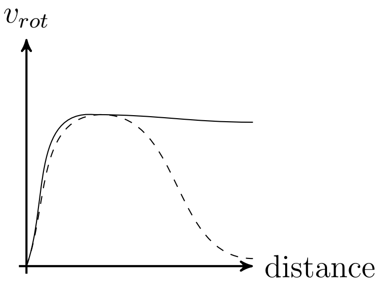
   - Galaxies cluster together. We can see how galaxies are moving and by their velocity distributions we can predict how much matter is luminous.

2. Rotational curves

   - The basic idea is to look at the motion of a galaxy as it rotates (here we assume a spiral galaxy),

     You look at the motion of a spiral galaxy as its rotating. Because the amount of gravitating mass dictates the rotational velocity as you study objects that are increasingly distant from the galaxy itself should be moving more and more slowly. But if you actually plot the rotation velocity as a function of the distance from the center of the galaxy we find,

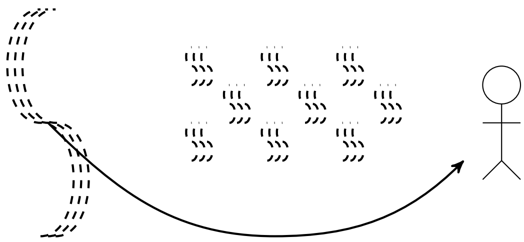
     

where the dashed line is the expected distribution and the solid line is what is found in practice. These are known as "rotational curves". We say that we have flat rotation curves.

This is an important observation because its telling you that we have mass out there that we can't see but also that the mass is distributed in a different fashion from ordinary matter. If we just had extra mass but still of the same type then we would have a different normalization of the curves above but not a different distribution. We can conclude that there is mass far away from the center pulling the objects. This is referred to as the halo of the galaxy.

3. Lensing

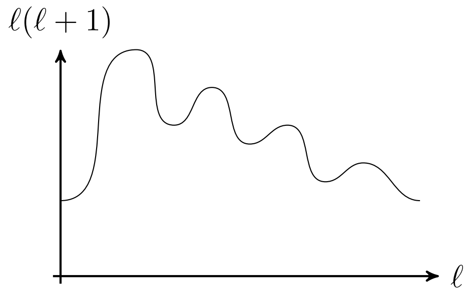
   - The situation is you would have some distance object (e.g. galaxy or quaser). Furthermore, you have some galaxy cluster in between you and the object. Then the light from the distant object will bend to you,



   The degree to which light is deflected and distorted can be used as a measure of the total mass, much of which is found to be unobservable.

4. CMB

   - We know the plot of the acoustic peaks of the CMB [Q 3: What's the y axis?],



   The different peaks tell you about how the fluid in the early universe is moving around. In particular, baryons which are charged couple significantly to the photon bath while DM does not couple very much. Therefore, the properties of the CMB can be used to infer the total amount of matter but also that most is not charged.

5. BBN

- Studies of primordial abundances of light elements tells us that,

$$\frac{n_{baryon}}{n_\gamma} \sim 6 \times 10^{-9} \tag{13}$$

which implies that the total relative density of baryonic matter is about $\Omega \sim 0.04$.

What people will usually say about DM is that it is cold and collisionless, cold dark matter (CDM), or collisionless cold dark matter (CCDM). Lets now go over what each of these words mean.

1. Matter

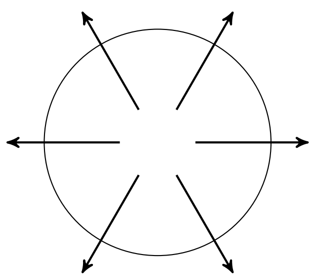
   - The energy density of a fluid expands as, $\rho_{DM} \propto a^{-3(w+1)}$ (where w is the equation of state parameter). w deviates from 0 when we have interesting interactions. The usual statement is that for DM $w \approx 0$. This is usually the zeroth order thing to require from DM.

2. Dark

   - This it the requirement that it doesn't couple to light. There are a lot of constraints on this. The biggest constraint arise from the CMB which tells you that DM should have a charge small enough that it is not in kinetic equilibrium with a thermal bath at $T \sim$ eV. This is a mass dependent and theory dependent statement [1].

3. Cold

   - This says that the DM is non-relativisitic at some era. This is important because you can often hear about people talking about hot or warm DM. However, even for DM that was relavistic at some point is still non-relativistic. All DM models are warm today. If DM is hot then you have some overdensity of DM and hot DM will fly out,



   This smooths out overdensities which will occur until the particle slows down and becomes cold. This tells you that when you look at the scales in the sky there will be some distance scale where you won't see very much structure.

[1] See Dubovsky,Garbanov, Rubtsov at arXiv:031189, for more details.

This is not observed. We currently see structures which are consistent with primordial fluctuations down to every scale we can observe. This means that DM should have been cold at $T \sim mboxkeV$ or earlier.

4. Collisionless

   - This is usually the requirement is just the universe should not have had a DM on DM scattering on average. This is usually enforced by the constraint that,

     $$n_\chi \sigma v \tau = 1 \tag{14}$$

     where τ is the lifetime of the universe. Applying this to some charactoristic system like the Milky way gives,

     $$\frac{\sigma}{10^{-24}\mathrm{cm}^2} \lesssim \frac{\mathrm{TeV}}{m_{DM}} \tag{15}$$

     This is fairly big interaction so DM can have very sizable interaction.

4 Models of dark matter

Now we go over the different models of dark matter

Name	What is it?	Motivation	Comment
axion	$\left(\bar{\theta} + \frac{a}{f}\right) g_{\mu\nu}\tilde{g}^{\mu\nu}$	strong cp problem	
neutralino	$\tilde{b}, \tilde{w}_3, \tilde{h}_u, \tilde{h}_d$	hierarchy problem	
sneutrino	$\tilde{\nu}$	hierarchy problem	dead if conventional
sterile ν	kev mass, small yuk	minimality	squeezed (must be $\sim$ kev)
ltop	lightest t-odd particle	hierarchy problem	common in little higgs
kkdm	kaluza klien dm tev xd	why not?	
axiono	$\tilde{a}$ superpartner of a	strong cp/heiararchy	
gravitino	$\tilde{g}$	heiararchy problem	
inert doublet	$(2 \pm 1/2)$	simple	

5 Forming DM

Very loosely there are two ways to form DM matter, thermal and non-thermal. Thermal means that DM was in equilibrium. Non-thermal productions mechanisms can be decay of another particle, phase transitions, or incomplete thermalization (processes that produce DM but don't get far enough along to be in thermal equilibrium).

5.1 Canonical example 1: oscillating scalar field

Consider the Klien Gordan equation for homogenous field in the expanding universe,

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \tag{16}$$

[Q 4: Show this.] For our purposes we take H to constant. That's not actually the case in our universe but it doesn't change the results we discuss here.

We can solve the differential equation by guessing, $\phi = \phi_0 e^{\omega t}$. Inserting into the above we have,

$$\omega^2 + 3\omega H + m^2 = 0 \tag{17}$$

which has the solutions,

$$\omega = \frac{-3H \pm \sqrt{9H^2 - 4m^2}}{2} \tag{18}$$

We can then take the limiting cases.

For $m \ll H$ we have two solutions [Q 5: check],

$$\omega = -3H, \frac{-2m^2}{3H} \tag{19}$$

The total energy density of this field is[2],

$$\rho \sim \dot{\phi}^2 + m^2\phi^2 \tag{20}$$

For $\omega = -3H$ the energy density scales as,

$$\rho \propto e^{-6Ht} \sim a^{-6} \tag{21}$$

where we used the fact that for $H = const$, $a \sim e^{Ht}$. So for the first solution the energy density decays extremely rapidly. For the second solution we find,

$$\phi^2 \sim m^2 \exp\left(-\frac{2m^2}{3H}\right) \tag{22}$$

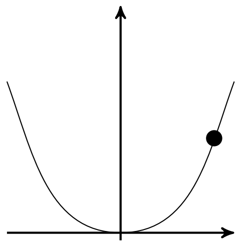
but by assumption, $m \ll H$, so,

$$\phi^2 \sim m^2 \to const \tag{23}$$

So if you have a mass much lighter then H then we have a part thats just going to sit there and not do anything.

Schematically what this mode looks like you have some potential and the field will just sit there and not move,

[2]Recall that the total energy density is what we normally refer to as the hamiltonian in field theory



Now lets consider the opposite limit, $m \gg H$. Then the solutions are,

$$\omega = -\frac{3H}{2} \pm im \tag{24}$$

The energy density goes as,

$$\rho \sim m^2 e^{-3Ht} \sim a^{-3} \tag{25}$$

The energy density is decaying with the volume which is how DM is known to behave. The history of the field is that it starts of with $H \gg m$ until it gets to the point that $m > H$ at which point it starts behaving like DM.

While this looks promising, this is generally speaking not a very good dark matter candidate. The reason is that since the energy density in the field is,

$$\rho \sim m^2 \phi^2 \tag{26}$$

The initial energy density should be something like,

$$\rho \sim m^2 M_{Pl}^2 \tag{27}$$

So we can ask when does the transition from $m < H$ to $H < m$? That is when $H^2 = m^2$ but the Hubble constant is ρ_{tot}/M_{Pl}^2. So this transition happens when,

$$\rho_{tot} = m^2 M_{Pl}^2 \approx \rho_\phi \tag{28}$$

This is bad because you go through this long period of the universe where the field does not act like dark matter. Eventually it starts oscillating, but as soon as it starts oscillating it basically is all the energy density of the universe. But we know when DM took over as the dominant contribution to the energy density of the universe, $\sim$ 1eV. So if this started dominating at 1eV then all the properties of DM happening prior to 1eV would not be present in this model, contradicting what we see in Nature [Q 6: What is he talking about?].

But the important thing to emphasize is that this oscillating scalar field is a good candidate for dark matter. Very loosely, this is a model for the axion.

Axions come about to solve the strong CP problem. We have the term, $\bar{\theta} G_{\mu\nu} \tilde{G}^{\mu\nu}$, where $\bar{\theta}$ is found to be very small. To solve this problem you can promote $\bar{\theta}$ to a field and given it a small VEV. [Q 7: Is this the correct interpretation?] The term becomes,

$$\left(\bar{\theta} + \frac{a}{f} \right) G_{\mu\nu} \tilde{G}^{\mu\nu} \tag{29}$$

where f is a charactoristic dimensionful scale for the axion, known as the axion decay constant. The fact that the axion interacts to $G_{\mu\nu}\tilde{G}^{\mu\nu}$ leads to a non-trivial potential for the axion,

$$V = m_\pi^2 f_\pi^2 \left[1 - \cos\left(\bar{\theta} + \frac{a}{f}\right)\right] \tag{30}$$

[Q 8: check this potential, where did it come from?] So at very high temperatures, before QCD becomes strongly interacting, the axion is essentially massless. After the QCD phase transition you generate a potential for the axion. The axion will be stuck at some particular field value once this potential turns on and it will begin to oscillate. It will then begin to act as a form of DM. You can calculate the mass of the axion by expanding the potential,

$$m_a \sim 6 \times 10^{-6}\mathrm{eV} \times \frac{10^{12}\mathrm{GeV}}{f} \tag{31}$$

For a $\mathcal{O}(1)$ initial value of the axion we have,

$$\Omega_a h^2 = 0.7 \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{7/6} \left(\frac{\theta_i}{\pi}\right)^2 \tag{32}$$

where, h is the hubble constant in units of 100km/(sec Mpc) and is roughly given by 0.7. The axion is massless up to its effects of QCD. The massless field can arise from a Goldstone boson from the breaking of a Pecce Quinn (PQ) symmetry. The θ_i parameterizes the initial conditions of the axion field when this symmetry breaking occurs.

If you start with an axion in the early universe with a variable decay constant there will be different amounts of energy density in DM today and its set by this formula. If $f_a \sim 10^{12}$GeV then the dark matter density is roughly of the correct order. Apriori the decay constant for the axion can be anything.

The axion model is an interesting example of how you get a good top-down model for dark matter. You have a problem, the strong CP problem. This problem tells you that the coefficient of the $G_{\mu\nu}\tilde{G}^{\mu\nu}$ should be zero (or almost zero). You introduce some dynamical field to cancel it off and then you study dynamics of that field. The dynamics of the field are that it should start at some value and start oscillating around it.

If axions exist then they usually also have coupling to photons. In this case they can be produced by stars and should contribute to their cooling. This puts a bound on the constant of axions of,

$$f \gtrsim 10^9\mathrm{GeV} \tag{33}$$

So there is still a lot of parameter space left.

If the PQ symmetry is broken after inflation then the value the field will roll off to will be random in each horizon on the universe. So you'll have on order roughly π value for the axion, i.e.,

$$\frac{f}{a} \sim \pi \tag{34}$$

[Q 9: I think this is equal to θ_i for whatever reason.] Then you roughly have the right relic density for the DM.

But if the symmetry is broken after inflation, then θ_i is one value in the whole universe. This value of θ_i will then get inflated and in principle it can be very small. This allows a much larger region of f_A that still satisfies the relic density constraints.

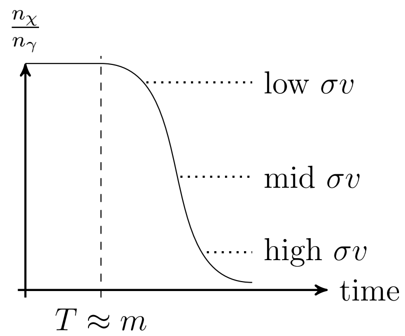
6 Thermal dark matter candidates

The idea here is there is some process in the early universe that converts ordinary matter into dark matter and vice versa. The way this is going to work is this is parameterized by some annihilation cross section, σv. This will tell us how efficiently dark matter converts into SM.

Recall that the number density for a non-relativistic particle scales like,

$$n_\chi \sim (m\tau)^{3/2} e^{-m/\tau} \tag{35}$$

So as time progresses we have,



If this system stayed in equlibrium the DM would continue to annihilate until there was no DM at all. However, this won't happen because eventually you'll get to a situation where there are not enough DM particles around to find each other to annihilate. So if you have a very low cross section at one point you will break off of equilibrium.

So for some value of σv we would expect to have an appropriate amount of DM left by tuning the cross-section.

The process of the system being in equilibrium and then stopping to be in equilibrium is called freezedown. The condition for freezeout is that,

$$n_\chi \langle \sigma v \rangle = H \tag{36}$$

The left hand side is the rate that any dark matter particle floating around in the universe annhilates with another DM particle. This is the condition for freeze out since if DM has not found another DM particle to anniihilate with in one hubble time at that point the universe has essentially doubled in size. At this point the universe is much more dilute and the probability of having any more annihilations is very small.

A simple observation is to note that the temperature of the freezeout is equal to the mass of the DM matter,

$$T_{FO} = x m_\chi \tag{37}$$

where x is some order 10 number. The reason for that is due to the exponential in the scaling of the number density. [Q 10: sharpen this.]

We can solve this becuase we have,

$$n_{FO} = \frac{H}{\sigma v} = \frac{T^2}{M_{Pl}\sigma v} = \frac{m_\chi^2 x^2}{M_{Pl}\sigma v} \tag{38}$$

That's the number density at freezeout. What you actually want to know is the number density today. Recall that the temperature scales as, $T \sim a^{-1}$. So we can write,

$$n_{now} = n_{FO}\left(\frac{a_{FO}}{a_{now}}\right)^3 = n_{FO}\left(\frac{T_{now}}{T_{FO}}\right)^3 \tag{39}$$

So we finally have,

$$n_{now} = \frac{m_\chi^2 x^2}{M_{Pl}\left\langle\sigma v\right\rangle}\frac{T_{now}^3}{m_\chi^3 x^3} \tag{40}$$

with,

$$\rho_{now} = \frac{T_{now}^3}{M_{Pl}\left\langle\sigma v\right\rangle x} \tag{41}$$

The energy density now is just dicated by the photon temperature, x, v, and the cross-section. The important observation is that the energy density is determined by σv (up to order 1 factors we haven't mentinoed here).

There is a simple way to calculate the cross-section that you need. There is the temperature of matter radiation equality. This is the tempeature where DM had the same energy density as radiation. This occured at about 1eV. Instead of asking what is the temperature now we can ask what is the temperature of matter radiation equality. Using the expression above we have,

$$\rho_{MRE} = T_{MRF}^4 = m_\chi n_{FO}\frac{T_{MRE}^3}{T_{FO}^3} \tag{42}$$

$$\rho_\gamma = \frac{m_\chi x^2 m_\chi^2}{M_{Pl}\sigma v}\frac{T_{MRE}^3}{x^3 m_{\chi^3}} \tag{43}$$

which implies that,

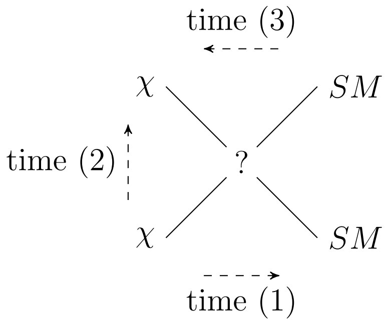
$$\frac{T_{MRE}^3}{M_{Pl}\sigma v x} = T_{MRE}^4 \tag{44}$$

This allows you to solve for this cross section,

$$\sigma v = \frac{1}{x M_{Pl} T_{MRE}} \tag{45}$$

This quantity is about the TeV scale. This is referred to the Weakly Interacting Massive Particle (WIMP) miracle. Particles with weak scale cross-sections end up giving the appropriate amount of DM.

This a very well known model, partly because its very detectable. If this is the process that keeps you in thermal equilibrium in the early universe,



then you've just measured the size of this to be, $\sigma v \sim 1/M_W^2$. What you've measured is the size of the diagram going in the horizontal direction. One can then argue that if it exists in the horizontal direction it should also exist in the other directions. This gives us three ways to probe WIMPs. The different ways are known as follows,

(1) Indirect detection: $\chi\chi$ annihilation in the halo

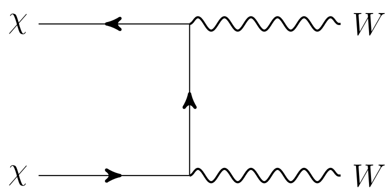
(2) Direct detection: undergraduate experiment where you hold the DM particle will come through and recoil onto a nucleus (this assumes the interaction involves a DM interaction with quark or gluon).

(3) Colliders: Look for missing energy processes at the LHC.

7 Canonical WIMP

In thinking about WIMPs, its good to have in mind a canonical model. We take this model to be a fermion doublet under $SU(2)$ with hypercharge $\pm 1/2$. This is recognized as a pure Higgsino or a right handed neutrino.

The first thing you do when studying the model of a WIMP is to ask how is it freezing out. For such a particle the dominant annihilation channel is,



The cross section is given by,

$$\langle \sigma v \rangle = \frac{g^4}{512\pi m_\chi^2}(21 + 3\tan\theta_w^2 + 11\tan\theta_w^4) \tag{46}$$

[Q 11: calculate this.] Note that this in velocity independent. This is because you do a partial wave expansion and the leading velocity term is the s wave peice and is velocity independent. The target cross section is, $\langle \sigma v \rangle = 3 \times 10^{-26} \mathrm{cm}^3/s$. You only have one free parameter here, the dark matter particle mass. The energy density is given by,

$$\Omega_\chi h^2 = 0.1 \times \left(\frac{m_\chi}{\mathrm{TeV}}\right)^2 \tag{47}$$

So we see that we need $m_\chi \sim$ TeV (about an order of magnitude above the weak scale).

Now lets explore whether this is ruled out through direct detection. As a homework exercise one can show that the differential energy rate of scattering of a dark matter particle for a spin-independent scattering,

$$\frac{dR}{E_R} = N_T M_N \frac{\rho_\chi \sigma_n}{2 m_\chi \mu_{ne}^2} \left(f_p z + f_n (A - Z)\right)^2 F^2(E_R) \times \int_{v_{min}(E_R)}^{\infty} \frac{f(v)}{v} dv \tag{48}$$

[Q 12: What exactly is R?] The variables in this expression are,

- N_T: number of targets
- M_N: mass of the nucleous
- ρ_χ: local dark matter density ($\approx 0.3 \mathrm{GeV}/\mathrm{cm}^3$)
- σ_n: cross section per nucleon
- μ_{n_e}: is the reduced mass of the WIMP-nucleon system (roughly equal to m_{proton})
- f_p and f_n are the couplings to the proton and neutron
- A, Z is the atomic mass and atomic number of the nucleus
- $F^2(E_R)$ is a form factor distribution
- $f(v)$ is the speed distribution. The usual assumption that is made is that the DM has some Bolzmann distribution, which gives you $f(v)$.

It has been show that, [3]

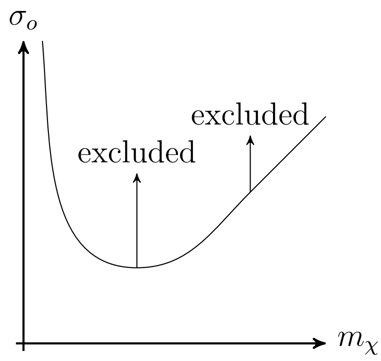
$$\sigma_{tot} = \frac{G_F^2}{2\pi} \mu_\chi^2 \left((1 - 4\sin^2\theta_w) Z - (A - Z)\right)^2 \tag{49}$$

You now have to do a backwords thing. You are really interested in comparing with the exclusion plots, which show the cross-section per nucleon. This is given by,

$$\sigma_o = \frac{G_F^2}{2\pi} \mu_{\chi n_e} \frac{[\ldots]}{\Lambda^2} \approx 2 \times 10^{-39} \mathrm{cm}^2 \tag{50}$$

The limits are roughly given as follows:

[3] See a paper by Rouven Essig, 0710.1667.



Any given experiment has some minimum mass for which they can see recoil. So for very light WIMPs it becomes exponentially unlikely to detect the WIMPs. At high masses since you know the energy density, $n_\chi = \rho_\chi / m_\chi$ the number density gets smaller and smaller, while ρ_χ is still constant. The current best limit has a minimum value of 50GeV and a exclusion cross-secton down to $2 \times 10^{-45}\mathrm{cm}^2$ (from the XENON100 experiment). The model of DM we discussed above is completely ruled out by this result.

7.1 Neutralino

For the neutralino you start with an antisymmetric mass matrix,

$$M_\chi = \begin{array}{c} \begin{array}{cccc} \scriptstyle\tilde{B} & \scriptstyle\tilde{W} & \scriptstyle\tilde{H} & \scriptstyle\tilde{H} \end{array} \\ \begin{bmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_w & m_Z \sin\beta \sin\theta_w \\ & M_2 & m_Z \cos\beta \cos\theta_w & -m_Z \sin\beta \cos\theta_w \\ & & 0 & -\mu \\ & & & 0 \end{bmatrix} \end{array} \tag{51}$$

where $\tan\beta \equiv v_u/v_d$ the ratio of the Higgs VEVs. What you used to have in supersymmetry is that we expect $M_1 \sim M_2 \sim \mu$, in which case this is a very mixed up matrix. However, as time has gone on and limits on these quantities were placed, you often get a much more block diagonal matrix. If the bottom right block produces a pure Higgsino then we almost have an $SU(2)$ doublet with $\pm 1/2$ hypercharge.

Naively we expect such a particle is excluded due to the calculation above, where we found the cross section was about 6 orders of magnitude stronger then the existing limits. However, this is not the case. This is because the scattering cross-section we calculated does not exist for this particle. The scattering operator we have is Z exchanged. The operator looks like,

$$\bar{\chi}\gamma^\mu \chi \bar{n}\gamma_\mu n \tag{52}$$

But the key point is that $\bar{\chi}\gamma^\mu\chi$ will vanish if χ is a Majorana fermion. Therefore, the dominant interact is actually Higgs exchange which is several orders of magnitude smaller.

8 Anomalies

Loosely speaking we have both indirect and direct detection anomalies.

The indirect anomalies are as follows. The Fermi galactic center telescrope "bump" as well as the Fermi 130GeV line both arise from the galactic center. There is also an excess of integral 511keV line also from the galactic center. Finally we have the Pamela/AMS high energy positron local anomaly.

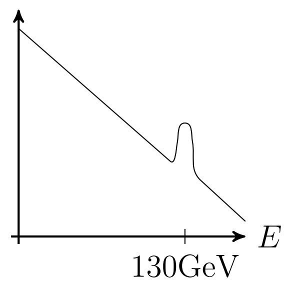
The direct anomalies are DAMA, which claims to see an annual modulation, CoGeNT, CoGeNT modulation, CRESST, CDMS-Si.

We now discuss some of these in some detail.

8.1 Indirect detection

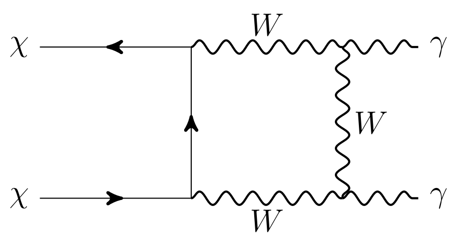
8.1.1 Fermi line

If you look at the spectrum in the galactic center you see a very nice spectrum until you get to about 130GeV where there is a sharp line,



This is obviously very interesting since its one of the smoking gun signitures of DM. The big concern is that the cross section is relatively high, $\sim 10^{-27}\text{cm}^3/s$.

More critically, people refer to the "continuum" constraints (see Lisanti et al). This says that if you look at the spectrum in the inner galaxy, you see this nice line but you don't see any deviations from a simple power law at lower energies. Most models that people tend to play with have something like this to produce light,

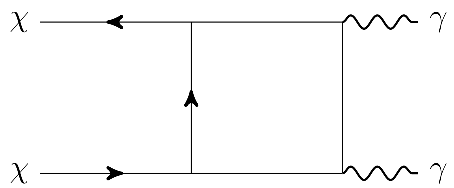


So the annihilation to photons should come with an annihilation to Ws. These should decay hadronically into π^0s with also give phtons. Therefore, we should get an additional bump at low energies which is not observed. You can place a limit on how large the observed ratio of the photon diagram to the WW diagram is. The limit is given by,

$$R = \frac{\langle \sigma v \rangle_{WW}}{\langle \sigma v \rangle_{\gamma\gamma}} < 10 - 100 \tag{53}$$

depending on whether you want to fit the expected shape of the bump or just counting excess photons. The problem is that this is a loop suppressed process, so should be suppressed by a atleast $16\pi^2$ and if you add couplings we need $R \sim 200 - 500$.

The solution is then to consider models with mediators that you can't decay into,

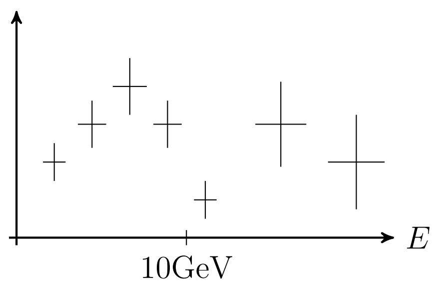


where the solid line is some heavy mediator. But to see the signal we still need this diagram to be significant so we must have

$$m_{med} \lesssim 300\text{GeV} \tag{54}$$

8.1.2 Fermi bump

The idea here is if you go into the galactic center there are various sources that you think should be there. There is a disk source, a steller halo, a point source, etc. You can subtract these things off and get a residual distribution that looks something like,



Note to get this plot you need to really understand the sources of light in the galactic center. This can be fit with either $m_\chi \sim 10$GeV with $\chi\chi \to \tau\tau$ or $m_\chi \sim 50$GeV with $\chi\chi \to b\bar{b}$. The cross section that you need is $\sigma v \sim 10^{-27}\text{cm}^3/s$ (for the lighter case).

The distribution requires a DM halo that scales like, $\rho \sim r^{-1.3\pm0.1}$ in the galactic center. This is very hard to explain using other light sources such as pulsers that we don't know about because pulsers should not have such a steep profile.

8.1.3 Dark force models

In the SM we have

$$SU(3) \times SU(2) \times U(1) \tag{55}$$

and every field has hypercharge, though not every field is charged under the non-abelian groups. I you can add to this some additional dark gauge group,

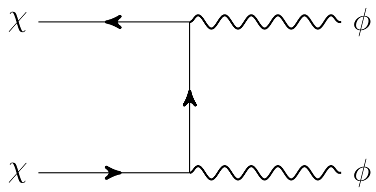
$$SU(3) \times SU(2) \times U(1) \times G_{dark} \tag{56}$$

The simplest case is that $G_{dark} = U(1)$. [Q 13: Why wouldn't all the SM particles be charged under this gauge group?] If this is true then we should also have a kinetic mixing term,

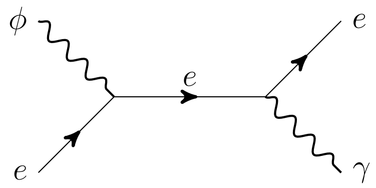
$$\mathcal{L} \supset \epsilon F'_{\mu\nu} F^{\mu\nu} \tag{57}$$

If the dark photon mass is roughly weak scale or higher then its appropriate to this of this as a mixing between the hypercharge. If the dark photon mass is below this point then you have EM mixing. As a homework exercise one can show that all SM particles acquire $q\epsilon$ charges under the G_{dark}. This allows you to have a lot of freedom in the phenomenology.

The early universe process of such models is dark matter annihilating into these dark photons, ϕ,



The dark photons stay in thermal equilibrium through a process such as,

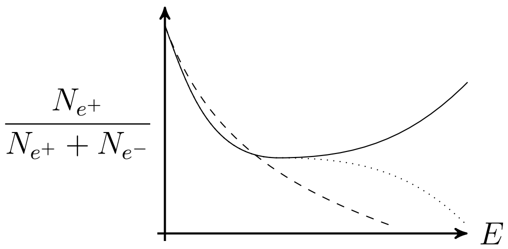


Even though the interaction of the dark photon is suppressed by the paramter ϵ, because you are competing with the hubble rate which is suppressed by the planck scale, this process can keep the dark photon in equilibrium.

The annihalation is then the dark matter annihilates into dark photons which will then propagate and decay. The dark photon decay modes will depend pretty much just on its mass. For $m_\phi \ll m_\rho$ we only have $\phi \rightarrow e^+e^-, \mu^+\mu^-$. However, as $m_\phi \gtrsim m_\rho$ we also have $\phi \rightarrow hadrons$. In this case we will also have π^0s which will give you photons. Thus one explanation of the fermi bump dark matter decaying into dark photons which decay into hadrons.

8.1.4 PAMELA and AMS

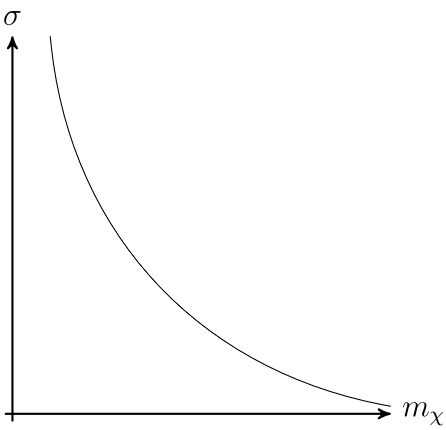
These dark photon models became popular in the context of Pomela and AMS which show a local excess of positrons. Ordinary astrophysical processes are expected to produce a fall in the number of positrons at high energies. The expected signal assuming no dark matter (dashed), the expected with most DM models (dotted), and the measured signal (solid) are shown below,



So to get this large signal you need a model that gives a hard spectrum of positrons and not give any associated hadronic modes which aren't observed. The dark force model can produce this shape for the signal.

8.2 Direct detection

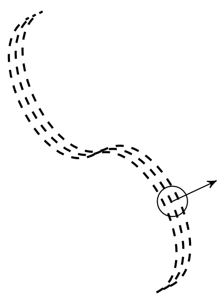
[Q 14: Insert direct detection plot from a paper...]



This is the current landscape of direct detection anomalies.

8.2.1 DAMA

DAMA is a sodium iodide experiment. They look for an annual modulation in count rate. The idea is that we are at some point far in the milky way,

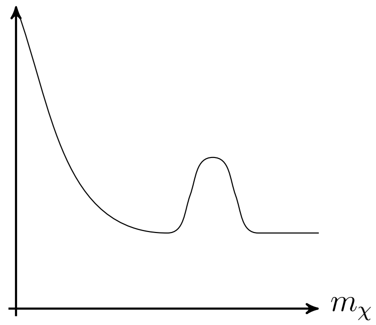


Our solar system is rotating around the galactic center, but the Earth is also moving around the Sun. Because of the motion of the Earth there is a dark matter wind. Furthermore, sometimes we move into the wind and sometimes we move away from the wind. Thus we expect a peak WIMP signal at one point in the year and a trough in another. This experiment has been running for years and they by now have a 9σ effect.

The usual reaction to this is that many things modulate such as cosmic rays. So its not clear whether they are really seeing a dark matter excess or something else.

8.2.2 CoGeNT

Is a Germanium experiment and they saw a spectrum of events that looks like,



The peak and the flat leveling off the signal are well understood from the properties of Germanium. The low energy effect was thought to initially not thought to be accounted for. Recently, it has been claimed that 70% of the rise at low energies can be accounted for by surface effects of the detector.

8.2.3 CRESST

CRESST is a confusing experiment which uses Calcium Tungstate detector. Depending on the target (Calcium, Tungstun, or Oxygen) different amounts of energy get deposited on their detector. They claim to see a slight excess as well.

8.2.4 CDMS-Si

This experiment has both Germanium and Silicon targets. Germanium is heavier and they have more of it so its thought to be a better target to look for dark matter. The silicon is primarily there to use as a cross-check. That was the initial thinking of the experimentalists. However, theorists were pushing for the collaboration to release their silicon data as its a lighter targer and is expected to be better at probing lighter WIMPs. They saw 3 events with an expectation of about 0.7.

8.3 What can we conclude?

All these anomalies point to a dark matter mass that is ruled out by the direct detection exclusion curves from Xenon100 . Thus there are three options,

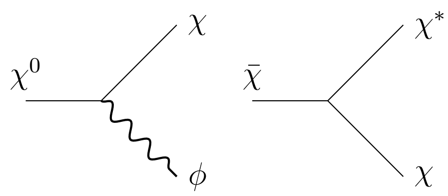
1. Experiments are wrong
2. Astrophysics is wrong
3. Model is wrong

There are a few ways to bypass the exclusion curves but still produce anomalies,

- Inelastic endothermic scattering
  - Favors heavy targets
- Inelastic exothermic scattering
  - Favors light targets
- Momentum dependent interactions
  - Favors experiments with high thresholds
- Spin dependent interactions
  - Favors targets with spin
- Isospin dependent interactions ($f_p \neq f_n$)
  - Kill 1 experiment
- Electron scattering
  - Favors non-descriminitory experiments

These tricks will favor one experiment over another one and shift the curves above which assume you can compare all the experiments. .

As an example we focus on inelastic exothermic interaction using a dark photon. If you have a dark photon then that photon should couple to a Dirac fermion. But ones the gauge symmetry is broken the two states can become pseudodirac which can be split into χ^* and χ,



[Q 15: Why does this shift the Xenon100 exclusion?]