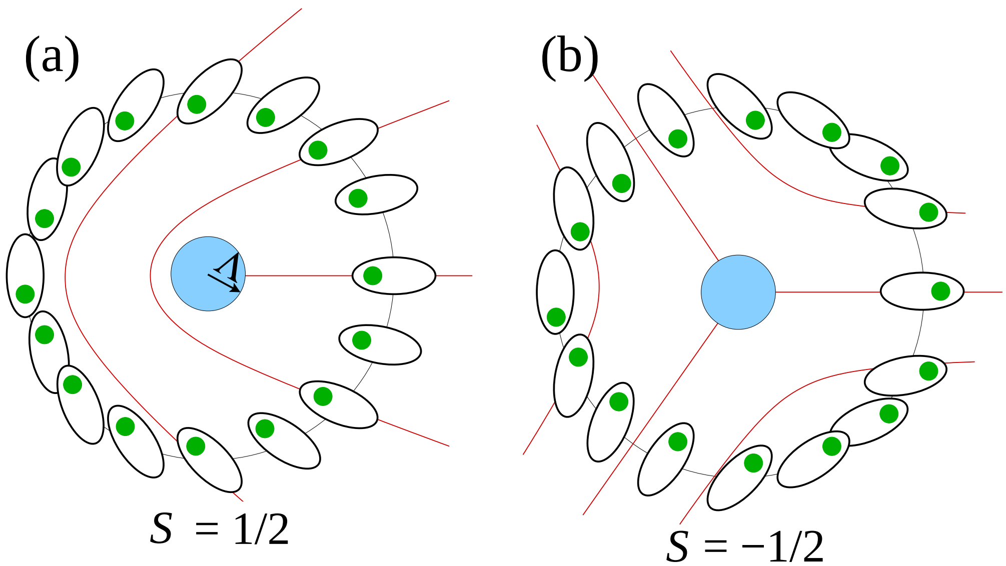
Teaser Question
Basic Training in Condensed–Matter Theory
James Sethna; Due Wednesday, January 28, 2009
Last revised: January 22, 2009, 2:48 pm



Nematic disclinations. The continuum director $\hat{\mathbf{n}}$ describes the fluid outside a small core around each defect, say of radius Λ.

Total derivatives and defect energies
A nematic liquid crystal is usually described in terms of a 'headless' unit vector $\hat{\mathbf{n}} \equiv -\hat{\mathbf{n}}$, since the molecules align along their long axes with no preference for head or tail. There are three terms in the traditional theory allowed by symmetry, proportional to $(\mathrm{div}\,\hat{\mathbf{n}})^2$, $(\hat{\mathbf{n}} \cdot \mathrm{curl}\,\hat{\mathbf{n}})^2$, and $(\hat{\mathbf{n}} \times \mathrm{curl}\,\hat{\mathbf{n}})^2$. However, there is a fourth "24" term allowed by symmetry, analogous to the term derived in class for multiferroics:

$$\mathcal{F}_{24} = (1/2)K_{24}\nabla \cdot (\hat{\mathbf{n}} \cdot \nabla\hat{\mathbf{n}} - \hat{\mathbf{n}}\nabla \cdot \hat{\mathbf{n}}). \tag{1}$$

It is usually ignored because it is a total derivative: by Gauss's law it is equal to a surface term

$$F_{24} = \int \mathcal{F}_{24} dV = \int (1/2)K_{24}(\hat{\mathbf{n}} \cdot \nabla\hat{\mathbf{n}} - \hat{\mathbf{n}}\nabla \cdot \hat{\mathbf{n}}) \cdot dS. \tag{2}$$

However, the continuum theory does not describe the core of line defects such as the disclinations shown in the figure above. Part of the surface S is not at infinity – it is surrounding each disclination at some core size cutoff radius Λ.
Will the contribution of eqn 2 be zero, finite, or infinite as $\Lambda \to 0$? For extra credit, assuming the disclination has the form $\hat{\mathbf{n}} = (\cos S\theta, \sin S\theta, 0)$, what is the contribution of eqn 2 to the energy per unit length of the disclinations?

Answer in box above. Bring to class Wednesday.

Name: ____________________

My first publication ("Theory of the Blue Phase of Cholesteric Liquid Crystals," S. Meiboom, J. P. Sethna, W. F. Brinkman, and P. W. Anderson, *Phys. Rev. Lett.* **46**, 1216 (1981)) argued that in the *blue phases* of chiral nematics this extra term gave the $S = -1/2$ disclination a negative energy per unit length, stabilizing a network of disclination lines with a lattice constant on the order of that for blue light.