

**Teaser Question**  
**Basic Training in Condensed-Matter Theory**

James Sethna; Due Friday, January 23, 2009

Last revised: January 20, 2009, 3:07 pm

**Why are fluids incompressible?**

A one-component fluid has three conserved fields: the energy density  $\epsilon(\mathbf{r})$ , the mass density  $\rho(\mathbf{r})$ , and the momentum density  $\mathbf{g}(\mathbf{r})$ . But for fluid flows on human scales, one can usually ignore both the energy density  $\epsilon$  and the variations in the density  $\rho - \rho_0$ , leading to the Navier-Stokes equations. We argued in class that Navier Stokes is an expansion at small Mach numbers, where the velocities  $\mathbf{v} = \mathbf{g}/\rho$  in the fluid are low compared to the speed of sound  $c$ . This we argued was equivalent to the translational kinetic energy density  $\rho\mathbf{v}^2/2$  being low compared to the internal energy of the fluids involved.

*Why are Mach numbers small in day-to-day life? Is there a fundamental mechanical reason why beings which could run faster than the speed of sound in their atmosphere would not survive? What would happen if they collided at a velocity comparable to their internal speed of sound? If possible, relate this to the practical limits to the strength and toughness of biological and other materials.*

*Answer in box above. Bring to class Friday. Will be graded, in lieu of homework.*

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